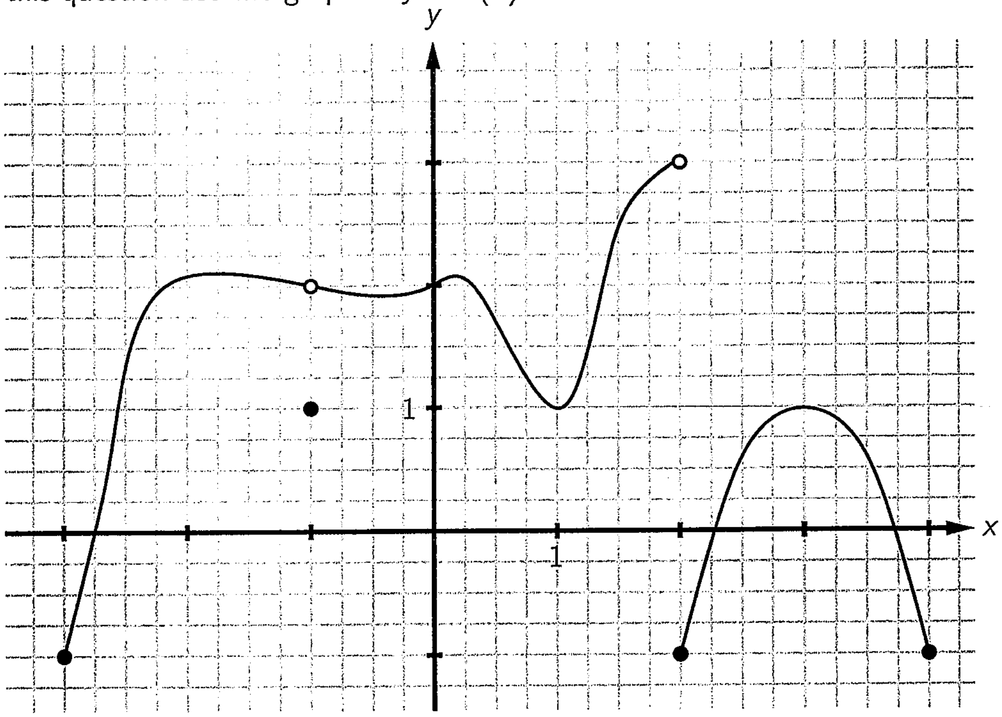
Question 1: For this question use the graph of $y = f(x)$ below:



(a)[2] What is $(f \circ f)(-3)$?

$f(f(-3)) = f(-1) = \boxed{1}$

(b)[1] State the range of $f(x)$ using interval notation.

$[-1, 3)$

(c)[1] State the domain of $f(x)$ using interval notation.

$[-3, 4]$.

(d)[2] Determine $\lim_{x\to 2} f(x)$.

limit does not exist.

(e)[2] What is $\lim_{x\to 2^-} f(x)$?

$\lim_{x\to 2^-} f(x) = \boxed{3}$

(f)[2] Determine $\lim_{x\to -1} f(x)$.

$\lim_{x\to -1} f(x) = \boxed{2}$

Question 2: For this question use the functions $f(x) = \dfrac{1}{x-1}$ and $g(x) = \sqrt{x-4}$.

(a)[3] Determine $(f+g)(x)$ and state the domain using interval notation.

$$(f+g)(x) = \frac{1}{x-1} + \sqrt{x-4}.$$

Domain: require $x-4 \geq 0$, $x \neq 1$

$\therefore$ $x \geq 4$

$\therefore$ $[4, \infty)$.

(b)[3] Determine $(f \circ g)(x)$ and state the domain using interval notation.

$$(f \circ g)(x) = \frac{1}{\sqrt{x-4}-1}$$

For domain: require $x-4 \geq 0$, $\sqrt{x-4} \neq 1$

$\therefore$ $x \geq 4$, $x \neq 5$

$\therefore$ $[4,5) \cup (5, \infty)$

(c)[4] Compute and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[\frac{1}{x+h-1} - \frac{1}{x-1}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{x}\cancel{-1}-\cancel{x}-h\cancel{+1}}{(x+h-1)(x-1)}\right]$$

$$= \boxed{\frac{-1}{(x+h-1)(x-1)}}$$

Question 3: Evaluate the following limits, if they exist:

(a)[3] $\lim_{x\to 3} \frac{x^2 - x - 6}{x^2 + 5x - 24}$ $\sim$ "$\frac{0}{0}$"

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+8)}$

$= \boxed{\frac{5}{11}}$

(b)[4] $\lim_{x\to 2^-} \left(\frac{1}{x-2} + \frac{1}{|x-2|}\right)$ $= \lim_{x\to 2^-} \left(\frac{1}{x-2} + \frac{1}{-(x-2)}\right)$

$= \lim_{x\to 2^-} 0$

$= \boxed{0}$

(c)[3] $\lim_{x\to 7^-} \frac{x - \sqrt{7-x}}{7+x}$ $= \frac{7-0}{14}$

$= \boxed{\frac{1}{2}}$

Question 4: Evaluate the following limits, if they exist:

(a)[5] $\lim_{x\to -3} \frac{\left[\frac{1}{3}+\frac{1}{x}\right]}{3+x}$ $\sim$ "$\frac{0}{6}$"

$= \lim_{x\to -3} \frac{1}{\cancel{3+x}}\left[\frac{\cancel{x+3}}{3x}\right]$

$= \boxed{\frac{-1}{9}}$

(b)[5] $\lim_{x\to 5} \frac{5-\sqrt{20+x}}{x-5} \cdot \frac{5+\sqrt{20+x}}{5+\sqrt{20+x}}$

$= \lim_{x\to 5} \frac{25-20-x}{(x-5)(5+\sqrt{20+x})}$

$= \lim_{x\to 5} \frac{5-x}{(x-5)(5+\sqrt{20+x})}$

$= \lim_{x\to 5} \frac{-\cancel{(x-5)}}{\cancel{(x-5)}(5+\sqrt{20+x})}$

$= \boxed{\frac{-1}{10}}$

Question 5:

(a)[3] Evaluate the following limit if it exists: $\lim_{\theta \to 0} \frac{\sin(3\theta)}{9\theta^2}$.

$$\lim_{\theta \to 0} \frac{\sin(3\theta)}{9\theta^2} = \lim_{\theta \to 0} \frac{\sin(3\theta)}{3\theta} \cdot \frac{1}{3\theta} \sim \text{"}\frac{1}{0}\text{"}$$

$\therefore$ Limit does not exist.

(b)[3] Evaluate the following limit if it exists: $\lim_{x \to 0} \frac{\sin(7x)}{\sin(2x)\cos(3x)}$

$$\lim_{x \to 0} \frac{\sin(7x)}{\sin(2x)\cos(3x)}$$

$$= \lim_{x \to 0} \left(\frac{\sin(7x)}{7x}\right) \cdot \frac{1}{\left(\frac{\sin(2x)}{2x}\right)} \cdot \frac{7x}{(2x)\cos(3x)}$$

$$= \boxed{\frac{7}{2}}$$

(c)[4] Suppose $f(x)$ is a function with the property that $-3 \le f(x) \le 2$ for every real number x. Determine $\lim_{x \to 0} x^4 f(x)$. (State any theorems used, like the the Squeeze Theorem, for example, and be sure to state the conditions necessary to justify use of the theorem.)

$$-3 \le f(x) \le 2$$

$$\therefore -3x^4 \le x^4 f(x) \le 2x^4$$

Since $\lim_{x \to 0} (-3x^4) = 0 = \lim_{x \to 0} (2x^4)$, by the Squeeze Theorem, $\lim_{x \to 0} x^4 f(x) = 0$.