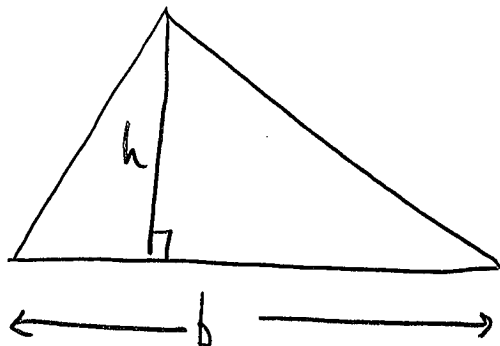


(1) [8] The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm²/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm²?



$$A = \frac{1}{2}bh.$$

$$\frac{dh}{dt} = +1 \frac{\text{cm}}{\text{min}}$$

$$\frac{dA}{dt} = +2 \frac{\text{cm}^2}{\text{min}}$$

What is $\frac{db}{dt}$ when $h=10$ and $A=100$?

$$\begin{aligned} \frac{dA}{dt} &= \frac{d}{dt} \left[\frac{1}{2}bh \right] \\ &= \frac{1}{2} \left[\frac{db}{dt} \cdot h + b \frac{dh}{dt} \right] \end{aligned}$$

$$\text{When } h=10, A=100, \quad b = \frac{2A}{h} = \frac{(2)(100)}{10} = 20,$$

$$\therefore 2 = \frac{1}{2} \left[\frac{db}{dt} \cdot 10 + 20 \cdot 1 \right]$$

$$\frac{db}{dt} = \frac{4 - 20}{10} = -\frac{16}{10} \frac{\text{cm}}{\text{min}}$$

\therefore base is decreasing at $\frac{16}{10} \frac{\text{cm}}{\text{min}}$.

(2) [3] Determine $\lim_{x \rightarrow 3^+} e^{2/(3-x)}$

As $x \rightarrow 3^+$, $3-x \rightarrow 0^-$, so $\frac{2}{3-x} \rightarrow -\infty$,

so $e^{\frac{2}{3-x}} \rightarrow 0$

$$\therefore \lim_{x \rightarrow 3^+} e^{\frac{2}{3-x}} = 0.$$

(3) [4] Determine the linear approximation (or linearization) $L(x)$ of $f(x) = 1/\sqrt{2+x}$ at $a = 0$.

$$f(x) = \frac{1}{\sqrt{2+x}}, \quad f(a) = f(0) = \frac{1}{\sqrt{2+0}} = \frac{1}{\sqrt{2}}$$

$$f'(x) = \frac{d}{dx} \left[(2+x)^{-\frac{1}{2}} \right] = -\frac{1}{2} (2+x)^{-\frac{3}{2}};$$

$$f'(a) = -\frac{1}{2} (2+0)^{-\frac{3}{2}} = -\frac{1}{2} \left(\frac{1}{2} \right)^{\frac{3}{2}} = \frac{-1}{4\sqrt{2}}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} x$$