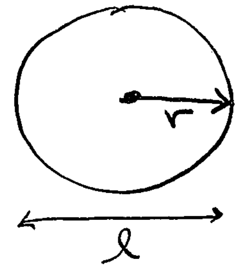
Math 121 - Quiz 5 Name: SOLUTIONS.

(1) [8] If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

(Recall: a sphere of radius r has volume $V = (4/3)\pi r^3$ and surface area $S = 4\pi r^2$.)



Let l = diameter $= 2r$.

$$\therefore \; S = 4\pi r^2 = 4\pi\left(\frac{l}{2}\right)^2 = \pi l^2$$

$$\therefore \; \frac{dS}{dt} = \frac{d}{dt}\left[\pi l^2\right] = \pi \, 2l \frac{dl}{dt}.$$

When $\frac{dS}{dt} = -1 \frac{\text{cm}^2}{\text{min}}$ and $l = 10$ cm:

$$-1 = \pi \cdot 2 \cdot 10 \cdot \frac{dl}{dt}$$

$$\Rightarrow \frac{dl}{dt} = \frac{-1}{20\pi} \frac{\text{cm}}{\text{min}}$$

$$\boxed{\therefore \text{ diameter decreases by } \frac{1}{20\pi} \frac{\text{cm}}{\text{min}}.}$$

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(2) [4] Determine the linear approximation (or linearization) $L(x)$ of $f(x) = 1/\sqrt{2+x}$ at $a = 0$.

$$f(x) = \frac{1}{\sqrt{2+x}} \;;\; f(a) = f(0) = \frac{1}{\sqrt{2}}$$

$$f'(x) = \frac{d}{dx}\left[(2+x)^{-\frac{1}{2}}\right] = -\frac{1}{2}(2+x)^{-\frac{3}{2}} \;;$$

$$f'(a) = f'(0) = -\frac{1}{2}(2)^{-\frac{3}{2}} = -\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{\frac{3}{2}} = -\frac{1}{2^{5/2}} = \frac{-1}{4\sqrt{2}}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$

$$\boxed{L(x) = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}}x}$$

(3) [3] Determine $\lim_{x\to 3^-} e^{2/(3-x)}$

As $x \to 3^-$, $3-x \to 0^+$, so $\frac{2}{3-x} \to +\infty$;

So $e^{\frac{2}{3-x}} \to \infty$

$$\boxed{\therefore \lim_{x\to 3^-} e^{\frac{2}{3-x}} = \infty.}$$