

Here are some additional problems that review material for the final exam. I've tried to include questions from all of the major topics we have done, but this is by no means a complete list of all the material you should know for the final exam.

1. Sketch the graph of $f(x) = \begin{cases} x^2 + 1 & \text{if } x < -1 \\ 3x + 1 & \text{if } x \geq -1 \end{cases}$ Find the following limits.

(a) $\lim_{x \rightarrow -1^-} f(x)$ (c) $\lim_{x \rightarrow -1} f(x)$
 (b) $\lim_{x \rightarrow -1^+} f(x)$ (d) $\lim_{x \rightarrow 1} f(x)$

2. Evaluate the following limits:

(a) $\lim_{x \rightarrow 0} (x^2 - 3x + 1)$ (f) $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$ (l) $\lim_{x \rightarrow 1} \frac{1-2x}{x^2-1}$
 (b) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$ (g) $\lim_{t \rightarrow -1} \frac{\frac{1}{2} + \frac{1}{t}}{2 + t}$ (m) $\lim_{x \rightarrow 2^+} \frac{x-4}{x^2 - 4x + 4}$
 (c) $\lim_{x \rightarrow 0} \frac{\sin x}{\tan x}$ (h) $\lim_{x \rightarrow 4^+} \sqrt{16 - x^2}$ (n) $\lim_{x \rightarrow 2^-} \frac{-x}{\sqrt{4 - x^2}}$
 (d) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$ (i) $\lim_{x \rightarrow -2^-} \sqrt{x^2 + 3x + 2}$ (o) $\lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{4 + x^2}}$
 (e) $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$ (j) $\lim_{x \rightarrow 1^-} \frac{1-2x}{x^2-1}$ (p) $\lim_{x \rightarrow \infty} \frac{x^3 - 2}{3x^2 + 4x - 1}$
 (k) $\lim_{x \rightarrow 1^+} \frac{1-2x}{x^2-1}$

3. Sketch the graph of a function that satisfies the following conditions:

- $f(0) = 0$
- $f'(-2) = f'(1) = f'(9) = 0$
- $\lim_{x \rightarrow \infty} f(x) = 0$
- $\lim_{x \rightarrow 6} f(x) = -\infty$
- $f'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$ and $(9, \infty)$
- $f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$
- $f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$
- $f''(x) < 0$ on $(0, 6)$ and $(6, 12)$

4. Given that $\lim_{x \rightarrow 0^+} \frac{1 - \cos x}{x^2} = \frac{1}{2}$, evaluate $\lim_{x \rightarrow 0^+} \frac{\sqrt{1 - \cos x}}{x}$

5. Use the fact that $\lim_{x \rightarrow a} f(x) = 2$, $\lim_{x \rightarrow a} g(x) = -3$ and $\lim_{x \rightarrow a} h(x) = 0$ to find the following:

(a) $\lim_{x \rightarrow a} [2f(x) - 3g(x)]$ (b) $\lim_{x \rightarrow a} \frac{f(x) + g(x)}{h(x)}$

6. Explain why each function is discontinuous at the given point.

(a) $f(x) = \frac{x}{x-1}$ at $x = 1$
 (b) $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$ at $x = 2$.

7. Find all discontinuities of $f(x)$. For each discontinuity that is removable, define a new function that removes the discontinuity.

(a) $f(x) = \frac{x-1}{x^2-1}$

(b) $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq -1 \\ x^2 + 5x & \text{if } -1 < x < 1 \\ 3x^3 & \text{if } x \geq 1 \end{cases}$

8. On what intervals is $f(x) = \sqrt[3]{x+2}$ continuous?

9. Determine all of the horizontal and vertical asymptotes of the following functions.

(a) $f(x) = \frac{x}{4-x^2}$

(c) $f(x) = \frac{3x^2+1}{x^2-2x-3}$

(b) $y = \ln(1 - \cos x)$

10. Suppose that the length of a small animal t days after birth is $h(t) = \frac{300}{1+9(0.8)^t}$ mm. What is the length of the animal at birth? What is the eventual length of the animal?

11. Suppose that $f(x)$ is a rational function $f(x) = \frac{p(x)}{q(x)}$ with the degree of $p(x)$ greater than the degree of $q(x)$. Does $f(x)$ have a horizontal asymptote?

12. Use implicit differentiation to find $\frac{dy}{dx}$.

(a) $x^2y^2 + 3y = 4x$

(c) $\sin(xy) + x^2 = x - y$

(e) $\sqrt{x+y} - 4x^2 = y$

(b) $\sqrt{xy} - 4y^2 = 12$

(d) $\frac{y}{x+1} - 3y = \tan x$

13. Use the definition of derivative to find

(a) $f'(2)$ for $f(x) = x^2 - 2x$

(c) $f'(x)$ for $f(x) = \frac{x^2+1}{x-2}$

(b) $f'(x)$ for $f(x) = x + \sqrt{x}$

14. Find the derivatives of the following functions.

(a) $y = x^4 - 3x^3 + 2x - 1$

(g) $y = \sqrt{\sin(4x)}$

(m) $y = \sqrt{\frac{x}{x^2+1}}$

(b) $y = \frac{3}{\sqrt{x}} + \frac{5}{x^2}$

(h) $y = \left(\frac{x+1}{x-1}\right)^2$

(n) $y = x \cos(5x^2)$

(c) $y = t^2(t+2)^3$

(i) $y = x^5\sqrt{x^3+2}$

(o) $y = \sin(\tan(x^2))$

(d) $y = \frac{x}{3x^2-1}$

(j) $y = \frac{x^3}{(x^2+4)^2}$

(p) $y = \frac{\sin(x^2)}{x^2}$

(e) $y = x^2 \sin x$

(k) $y = (\sqrt{x} + 3)^{4/3}$

(f) $y = \tan(\sqrt{x})$

(l) $y = (\sqrt{x^3+2} + 2x)^{-2}$

(q) $y = \tan(\sqrt{x^2+1})$

15. Find an equation of the tangent line.

(a) $y = x^4 - 2x + 1$ at $x = 1$

(b) $y - x^2y^2 = x - 1$ at $(1, 1)$

16. Find the indicated derivative

30. Use linear approximations to estimate $\sqrt[4]{16.04}$
31. Complete two iterations of Newton's Method to find a root of $x^4 - 3x^2 + 1 = 0$ starting at $x_0 = 1$.
32. Use Newton's Method to estimate a root, accurate to six decimal places of $\sin x = x^2 - 1$.
33. Use Newton's Method to estimate $\sqrt[44]{24}$.
34. Finding a root of $4x^3 - 7x^2 + 1 = 0$ with Newton's Method at $x_0 = 0$ fails. Explain why the method fails and if possible find a root by correcting the problem.
35. Find the critical numbers of the following functions.
- | | |
|---------------------------------|---|
| (a) $f(x) = x^4 - 3x^3 + 2$ | (d) $f(x) = x^{4/3} + 4x^{1/3} + 4x^{-2/3}$ |
| (b) $f(x) = x^{3/4} - 4x^{1/4}$ | (e) $f(x) = 2x\sqrt{x+1}$ |
| (c) $f(x) = \frac{x^2-2}{x+2}$ | (f) $f(x) = \frac{1}{2}(e^x + e^{-x})$ |
36. Find the absolute extrema of the given function on the given interval.
- | | |
|---------------------------------------|--|
| (a) $f(x) = x^3 - 3x + 1$ on $[0, 2]$ | (c) $f(x) = \frac{3x^2}{x-3}$ on $[-2, 2]$ |
| (b) $f(x) = x^{2/3}$ on $[-4, -2]$ | (d) $f(x) = e^{-x^2}$ on $[-3, 2]$ |
37. Sketch a graph of a function f such that the absolute maximum of $f(x)$ on $[-2, 2]$ is 3 and the absolute minimum does not exist.
38. Show that $f(x) = x^3 + bx^2 + cx + d$ has both a local minimum and a local maximum if $c < 0$.
39. For the family of functions $f(x) = x^4 + cx^3 + 1$ find all local extrema. (Your answer will depend on c .)
40. Find the intervals where the function is increasing and decreasing.
- | | |
|--------------------------|---------------------------|
| (a) $y = x^4 - 8x^2 + 1$ | (c) $y = \sin x + \cos x$ |
| (b) $y = (x + 1)^{2/3}$ | (d) $y = e^{x^2-1}$ |
41. Find all critical numbers and use the First Derivative Test to classify each as a local maximum or local minimum.
- | | |
|---------------------------|-----------------------------|
| (a) $y = x^4 + 4x^3 - 2$ | (c) $y = \sqrt{x^3 + 3x^2}$ |
| (b) $y = \frac{x}{1+x^3}$ | (d) $y = xe^{-2x}$ |
42. Find all asymptotes and extrema of $y = \frac{x^2}{x^2-4x+3}$.
43. Sketch a graph of a function with the given properties: $f(3) = 0$, $f'(x) < 0$ for $x < 0$ and $x > 3$, $f'(x) > 0$ for $0 < x < 3$, $f'(3) = 0$, $f(0)$ and $f'(0)$ do not exist.

44. Suppose that the total sales of a product after t months is given by $s(t) = \sqrt{t+4}$ thousand dollars. Compute and interpret $s'(t)$.

45. Show that $f(x) = x^3 + bx^2 + cx + d$ is an increasing function if $b^2 \leq 3c$.

46. Determine the intervals where the function is concave up and concave down.

(a) $f(x) = x + \frac{1}{x}$ (c) $f(x) = x^{4/3} + 4x^{1/3}$

(b) $f(x) = \sin x - \cos x$ (d) $f(x) = xe^{-4x}$

47. Find all critical number and use the Second Derivative Test to determine all local extrema.

(a) $f(x) = x^4 + 4x^3 - 1$ (c) $f(x) = xe^{-x}$

(b) $f(x) = \frac{x^2-5x+4}{x}$

48. Sketch a graph with the following properties: $f(0) = 0$, $f(-1) = -1$, $f(1) = 1$, $f'(x) > 0$ for $x < -1$ and $0 < x < 1$, $f'(x) < 0$ for $-1 < x < 0$ and $x > 1$, $f''(x) < 0$ for $x < 0$ and $x > 0$.

49. Suppose $w(t)$ is the depth of water in a city's water reservoir at time t . Which would be better news at time $t = 0$, $w''(0) = 0.05$ or $w''(0) = -0.05$ or would you need to know that value of $w'(0)$ to determine which is better?

50. A three sided fence is to be built next to a straight section of river which forms the fourth side of a rectangular region. The enclosed area is to equal 1800 square feet. Find the minimum perimeter and the dimensions of the corresponding enclosure.

51. A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 feet of fencing available, what dimensions of the corral will maximize the enclosed area?

52. Find the point on the curve $y = x^2$ closest to the point $(0, 1)$.

53. Find the point on the curve $y = \cos x$ closest to the point $(0, 0)$.

54. A company's revenue for selling x (thousand) items is given by $R(x) = \frac{35x-x^2}{x^2+35}$. Find the value of x that maximizes the revenue and find the maximum revenue.

55. Suppose that group tickets to a concert are priced at \$40 per ticket if 20 tickets are ordered, but cost \$1 per ticket less for each extra ticket ordered up to a maximum of 50 tickets. (For example if 22 tickets are ordered the price is \$38 per ticket.) Find the number of tickets that maximizes the total cost of the tickets.

56. Completely analyze each function (according to the checklist for curve sketching) and then use the information collected to graph the function.

(a) $f(x) = x^5 - 2x^3 + 1$

(e) $f(x) = (x^3 - 3x^2 + 2x)^{2/3}$

(b) $f(x) = x + \frac{4}{x}$

(f) $f(x) = \frac{x^2+1}{3x^2-1}$

(c) $f(x) = \sqrt{x^2+1}$

(d) $f(x) = \sqrt[3]{x^3 - 3x^2 + 2x}$

(g) $f(x) = \frac{5x}{x^3-x+1}$

57. The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error in computing
- the volume of the cube
 - the surface area of the cube
58. Find two positive numbers whose product is 100 and whose sum is a minimum.
59. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.
60. A balloon is rising at a constant speed of 5ft/s. A boy is cycling along a straight road at a speed of 15ft/s. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?
61. Find a parabola $y = ax^2 + bx + c$ that passes through the point (1, 4) and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2 respectively.
62. Sketch the graph of a function f that is continuous on $[1, 5]$ and has the following properties:
- absolute maximum at 5
 - absolute minimum at 2
 - local maximum at 3
 - local minima at 2 and 4
63. An airplane flying west at 300 miles per hour goes over that control tower at noon. A second airplane at the the same altitude flying north at 400 miles per hour goes over the control tower one hour later. How fast is the distance between the airplanes changing at 2:00 pm?
64. Chris, who is 6 feet tall, is walking away from a street light pole 30 feet high at a rate of 2 feet per second.
- How fast is his shadow increasing in length when Chris is 24 feet from the pole?
 - How fast is the tip of his shadow moving?
65. Find the dimensions of an aluminum can that holds 40 cubic centimeters of water and that uses the least amount of material. Assume the can is a cylinder with a top and a bottom.
66. Show that $f(x) = 2x^3 + 1$ and $g(x) = \sqrt[3]{\frac{x-1}{2}}$ are inverse functions.
67. Determine if the function is one-to-one. If it is, find the inverse function.
- $f(x) = \sqrt{x-1}$
 - $f(x) = \frac{1}{x+1}$
 - $f(x) = \frac{1-2x}{1+x}$
 - $f(x) = x^4 + 2$
 - $f(x) = \sqrt{x^3 + 1}$
68. Graph $f(x) = \sqrt{x^2 - 2x}$. Find an interval on which $f(x)$ is one-to-one and then find the inverse of $f(x)$ on the restricted domain.
69. Is the following an invertible function? $f(t)$ is the income of a company as a function of time t .
70. Solve the following equations for x .

- (a) $\log_4(x+4) - 2\log_4(x+1) = \frac{1}{2}$ (e) $xe^{-2x} + 2e^{-2x} = 0$
 (b) $2^{x^2-1} = 32$ (f) $4\ln x = -8$
 (c) $\log_2(x+3) - \log_2 x = 2$ (g) $x^2 \ln x - 9 \ln x = 0$
 (d) $e^x(x^2 - 1) = 0$ (h) $e^{2\ln x} = 4$

71. Rewrite $\log_a(x^4 + 3x^2 + 2) + \log_a(x^4 + 5x^2 + 6) - 4\log_a \sqrt{x^2 + 2}$ as a single logarithm.

72. Assume that the given function has an inverse. Without solving for the inverse, find the indicated function values.

- (a) $f(x) = x^3 + 4x - 1$; $f^{-1}(-1)$ and $f^{-1}(4)$
 (b) $f(x) = \sqrt{x^3 + 2x + 4}$; $f^{-1}(4)$ and $f^{-1}(2)$

73. Evaluate the following limits.

- (a) $\lim_{x \rightarrow 0} \frac{xe^{-2x+1}}{x^2 + x}$ (c) $\lim_{x \rightarrow \infty} \ln(2x)$
 (b) $\lim_{x \rightarrow 0} \frac{1 - e^{2x}}{1 - e^x}$ (d) $\lim_{x \rightarrow 0^+} e^{-2/x}$
 (e) $\lim_{x \rightarrow \infty} e^{2x-1}$

74. Differentiate the following functions.

- (a) $y = e^{2x} \cos(4x)$ (h) $y = \ln(\cos x)$
 (b) $y = x4^{3x}$ (i) $y = e^{\sin(2x)}$
 (c) $y = 2e^{4x+1}$ (j) $y = \sin(\ln(\cos x^3))$
 (d) $y = 4^{-3x+1}$ (k) $y = \ln(\sin x^2)$
 (e) $y = \frac{x}{e^{6x}}$ (l) $y = \frac{\sqrt{\ln x^2}}{x}$
 (f) $y = \ln(x^3 + x)$ (m) $y = \ln(\sec x + \tan x)$
 (g) $y = x^3 \ln x$ (n) $y = \sqrt[3]{e^{2x} x^3}$

75. Use logarithmic differentiation to find $\frac{dy}{dx}$ for the following.

- (a) $y = \frac{\sqrt{x+13}}{(x-4)\sqrt[3]{2x+1}}$ (c) $y = (x^2)^{4x}$
 (b) $y = x^{\sin x}$ (d) $y = x^{\ln x}$
 (e) $y = x^{\sqrt{x}}$

Answers

1. (a) 2 (b) -2 (c) Does not exist (d) 4
 2. (a) 1 (d) $\frac{1}{4}$ (g) $-\frac{1}{2}$ (j) ∞
 (b) 5 (e) 2 (h) Does not exist (k) ∞
 (c) 1 (f) $\frac{1}{2}$ (i) 0 (l) does not exist

(m) $-\infty$

(n) $-\infty$

(o) 1

(p) ∞

3. Answers will vary

4. $\frac{1}{\sqrt{2}}$

5. (a) 13

(b) Does not exist

6. (a) $f(1)$ is not defined

(b) $\lim_{x \rightarrow 2} f(x) \neq f(2)$

7. (a) $x = 1$ and $x = -1$

(b) $x = 1$

8. $(-\infty, \infty)$

9. (a) $x = \pm 2, y = 0$

(c) $x = -1, x = 3, y = 3$

(b) $x = 2\pi n$

10. 30mm, 300mm

11. no

12. (a) $\frac{4-2xy^2}{3+2x^2y}$

(c) $\frac{16x\sqrt{x+y}-1}{1-2\sqrt{x+y}}$

(e) $\frac{\sec^2 x + \frac{y}{(x+1)^2}}{\frac{1}{(x+1)} - 3}$

(b) $\frac{y}{16y\sqrt{xy}-x}$

(d) $\frac{1-2x^2-y\cos(xy)}{x\cos(xy)+1}$

13. (a) 2

(b) $1 + \frac{1}{2}x^{-\frac{1}{2}}$

(c) $\frac{x^2-4x-1}{(x-2)^2}$

14. (a) $4x^3 - 9x^2 + 2$

(j) $\frac{-x^2(x^2-12)}{(x^2+4)^3}$

(b) $-\frac{3}{2}x^{-3/2} = 10x^{-3}$

(k) $\frac{2(\sqrt{x}+3)^{1/3}}{3\sqrt{x}}$

(c) $2t(t+2)^3 + 3t^2(t+2)^2$

(l) $\frac{-3x^2(x^3+2)^{-1/2}-4}{(\sqrt{x^3+2}+2x)^3}$

(d) $\frac{(3x^2-1)-6x^2}{(3x^2-1)^2}$

(m) $\frac{-x^2+1}{2\sqrt{x}(x^2+1)^{3/2}}$

(e) $2x \sin x + x^2 \cos x$

(f) $\frac{1}{2}x^{-1/2} \sec^2(\sqrt{x})$

(n) $\cos(5x^2) - 10x^2 \sin(5x^2)$

(g) $2 \cos(4x)(\sin(4x))^{-1/2}$

(o) $2x \cos(\tan(x^2)) \sec^2(x^2)$

(h) $\frac{-4(x+1)}{(x-1)^3}$

(p) $2x^{-3}(x^2 \cos(x^2) - \sin(x^2))$

(i) $\frac{x^4(13x^3+20)}{2\sqrt{x^3+2}}$

(q) $\frac{x}{\sqrt{x^2+1}} \sec^2(\sqrt{x^2+1})$

15. (a) $y = 2x - 2$

(b) $y = -3(x - 1) + 1$

16. (a) $12x^2 - 18x + 4$

(b) $2 \sec^2 x \tan x$

(c) $-3^{26} \sin(3x)$

17. $(0, 1), (4, -31)$

18. 1

19. The stock is losing value.

20.

(a) $y = -2x - 3$ (b) $y = -x - 1$ (c) -12 (d) -8

21. $\frac{1}{\sqrt{3}}$ meters per second

22. 1.22 feet per minute

23. $\frac{10}{9\pi}$ feet per second

24. (a) $\frac{3}{5}x^5 - \frac{3}{2}x^2 + C$ (d) $-2 \cos x + \sin x + C$ (g) $x - 3e^{-x} + C$

(b) $2x^{3/2} + \frac{1}{3}x^{-3} + C$ (e) $5 \tan x + C$

(c) $\frac{3}{2}x^{2/3} - 9x^{1/3} + C$ (f) $\frac{2}{5}x^{5/2} - \frac{16}{5}x^{5/4} + C$

25. $6x^2 + 2x + 3$

26. $3t - 6t^2 + 3$

27. $-3 \sin t + \frac{1}{2}t^2 + 3t + 4$

28. $f(x) = \frac{x^3}{6} - \frac{x^2}{2} + \frac{7}{2}x - \frac{7}{6}$

29. $3 + \frac{1}{3}x$

30. 2.00125

31. $\frac{5}{8}$

32. $-0.636733, 1.409624$

33. 1.059133

34. $f'(0) = 0, -0.3454, 0.4362, 1.6592$

35. (a) $0, \frac{9}{4}$ (c) $-2 + \sqrt{2}, -2 - \sqrt{2}$ (e) $\frac{-2}{3}, -1$

(b) $0, \frac{16}{9}$ (d) $-2, 1$

36. (a) minimum -1 , maximum 3 (c) maximum 0 , minimum -12

(b) maximum $2^{4/3}$, minimum $2^{2/3}$

37. Answers will vary.

38. $4b^2 - 12c > 0$ if $c < 0$

39. $c \geq 0$ there is one relative minimum; $c < 0$ there are two relative minima and one relative maximum.

40. (a) increasing on $(-2, 0), (2, \infty)$; decreasing on $(-\infty, -2), (0, 2)$

(b) increasing on $(-1, \infty)$; decreasing on $(-\infty, -1)$

(c) increasing on $(-\frac{3\pi}{4} + 2n\pi, \frac{\pi}{4} + 2n\pi)$; decreasing on $(\frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi)$

(d) increasing on $(0, \infty)$; decreasing on $(-\infty, 0)$

41.

- (a) $x = -3$ (local minimum); $x = 0$ (neither) (c) $x = -2$ (local maximum), $x = 0$ (local minimum)
- (b) $x = \frac{1}{\sqrt[3]{2}}$ (local maximum)
42. vertical asymptotes at $x = 1$, $x = 3$, horizontal asymptote at $y = 1$, local minimum at $x = 0$, local maximum at $x = \frac{3}{2}$
43. Answers will vary
44. $s'(t) = \frac{1}{2\sqrt{t+4}}$; rate of increase of sales
45. Show that $f(x)$ is increasing everywhere
46. (a) concave up on $(1, \infty)$, concave down on $(-\infty, 1)$
 (b) concave up on $(-\frac{3\pi}{4} + 2n\pi, \frac{\pi}{4} + 2n\pi)$; concave down on $(\frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi)$
 (c) concave up on $(-\infty, 0)$, $(2, \infty)$, concave down on $(0, 2)$
 (d) concave up on $(-\infty, \frac{1}{2})$, concave down on $(\frac{1}{2}, \infty)$
47. (a) $x = -3$ (local minimum), $x = 0$ (test fails)
 (b) $x = -2$ (local maximum), $x = 2$ (local minimum)
 (c) $x = 1$ (local maximum)
48. Answers will vary
49. We need to know $w'(0)$
50. 30 feet by 60 feet; the perimeter is 120 feet
51. 20 feet by 30 feet
52. $(\frac{1}{\sqrt{2}}, \frac{1}{2})$ or $(-\frac{1}{\sqrt{2}}, \frac{1}{2})$
53. $(0, 1)$
54. $x = 5$, $R(5) = 2.5$
55. 30
56. Graphs omitted
57. (a) 270 cm^3
 (b) 36 cm^2
58. 10, 10
59. 500, 125
60. 13 ft/s
61. $y = -\frac{2}{3}x^2 + \frac{14}{3}x$

62. Answers will vary

63. 471 miles per hour

64. (a) 0.5 feet per second

(b) 2.5 feet per second

65. The radius is 1.85 cm and the height is 3.7 cm

66. Show that $f(g(x)) = x$ and $g(f(x)) = x$

67. (a) $f^{-1}(x) = x^2 + 1$

(c) $f^{-1}(x) = \frac{1-x}{2+x}$

(e) $f^{-1}(x) = \sqrt[3]{x^2 - 1}$

(b) $f^{-1}(x) = \frac{1}{x} - 1$

(d) $f(x)$ is not one-to-one

68. $x \geq 0$, $f^{-1}(x) = \sqrt{x^2 + 1} + 1$

69. Not an invertible function.

70. (a) $\frac{1}{2}$, -2

(d) ± 1

(g) 1, 3, -3

(b) $\pm\sqrt{6}$

(e) -2

(h) ± 2

(c) 1

(f) $\pm\sqrt[4]{e^{-8}}$

71. $\log_a(x^4 + 4x^2 + 3)$

72. (a) 0, 1

(b) 2, 0

73. (a) e

(d) 0

(b) 2

(c) ∞

(e) ∞

74. (a) $8xe^{2x} \cos(4x) - 4e^{2x} \sin(4x)$

(i) $2e^{\sin(2x)} \cos(2x)$

(b) $4^{3x} + 3x4^{3x} \ln 4$

(j) $\frac{-3x^2 \sin(x^3) \cos(\ln(\cos x^3))}{\cos x^3}$

(c) $8e^{4x+1}$

(k) $\frac{2x \cos x^2}{\sin x^2}$

(d) $-3e^{-3x+1} \ln 4$

(l) $\frac{x^2(\ln x^2)^{-1/2} - \sqrt{\ln x^2}}{x^2}$

(e) $\frac{e^{6x} - 6xe^{6x}}{e^{12x}}$

(m) $\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

(f) $\frac{x^2+1}{x^3+x}$

(g) $3x^2 \ln x + x^2$

(n) $\frac{1}{3}(e^{2x}x^3)^{-2/3}[2x^3e^{2x} + 3x^2e^{2x}]$

(h) $\frac{-\sin x}{\cos x}$

75. (a) $-\frac{10x^2+219x-118}{6(x-4)^2(x+13)^{1/2}(2x+1)^{4/3}}$

(d) $x^{\ln x} \left(\frac{2 \ln x}{x}\right)$

(b) $x^{\sin x}(\cos x \ln x + \frac{\sin x}{x})$

(e) $x^{\sqrt{x}}(\frac{1}{2}x^{-1/2} \ln x + x^{-1/2})$

(c) $(x^2)^{4x}(4 \ln x^2 + 8)$