

Question 1 [10 points]: An inverted cone full of water has top diameter and height both equal to 10 cm. The water drains from the cone at a rate of $32 \text{ cm}^3/\text{min}$. At what rate is the water level dropping when the water level in the cone is 4 cm?

(Recall: The volume of a cone of height h and base radius r is $V = \pi r^2 h/3$.)

Question 2:

(a)[5] Use a linear approximation to estimate the value of $\ln(0.99)$. (Recall that $\ln(1)$ is a “nice” number.)

(b)[5] Determine the linearization $L(x)$ of $f(x) = \frac{\tan(x) - x}{x}$ at $a = \pi$.

Question 3: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3] $y = 2^{\sec x} - \frac{e^{-x^3}}{x}$

(b)[3] $f(x) = \ln(x \sin^2 x)$

(c)[4] $y = (\sqrt{x})^{x+1}$ (logarithmic differentiation may be helpful here.)

Question 4: For this question use the function $f(x) = (x^2 - 8)e^x$.

(a)[7] Determine the intervals of increase and decrease of $f(x)$. State a clear conclusion.

(b)[3] State the relative (or local) extreme values of $f(x)$.

Question 5: Suppose $f(x)$ has domain all real numbers and first derivative $f'(x) = \frac{x}{x^2 + 36}$.

(a)[7] Determine the intervals of concavity of the graph of $y = f(x)$. State a clear conclusion.

(b)[3] State the x -coordinates of the inflection points, if any. (note: you do not have enough information to give the y -coordinates.)