Math 121 - Test 3 Mar 29 2012

Question 1 [10 points]: An inverted cone full of water has top diameter and height both equal to 10 cm. The water drains from the cone at a rate of $36 \text{ cm}^3/\text{min}$. At what rate is the water level dropping when the water level in the cone is 6 cm?

(Recall: The volume of a cone of height h and base radius r is $V = \pi r^2 h/3$.)

Question 2:

(a)[5] Determine the linearization L(x) of $f(x) = \frac{x - \tan(x)}{x}$ at $a = \pi$.

(b)[5] Use a linear approximation to estimate the value of $\ln(0.9)$. (Recall that $\ln(1)$ is a "nice" number.)

Question 3: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3]
$$f(x) = \ln(x \sin^2 x)$$

(b)[3]
$$y = 2^{\sec x} - \frac{e^{-x^3}}{x}$$

(c)[4]
$$y = (\sqrt{x})^{x+1}$$
 (logarithmic differentiation may be helpful here.)

- **Question 4:** For this question use the function $f(x) = (x^2 3)e^x$.
- (a)[7] Determine the intervals of increase and decrease of f(x). State a clear conclusion.

(b)[3] State the relative (or local) extreme values of f(x).

Question 5: Suppose f(x) has domain all real numbers and first derivative $f'(x) = \frac{x}{x^2 + 25}$.

(a)[7] Determine the intervals of concavity of the graph of y = f(x). State a clear conclusion.

(b)[3] State the *x*-coordinates of the inflection points, if any. (note: you do not have enough information to give the *y*-coordinates.)