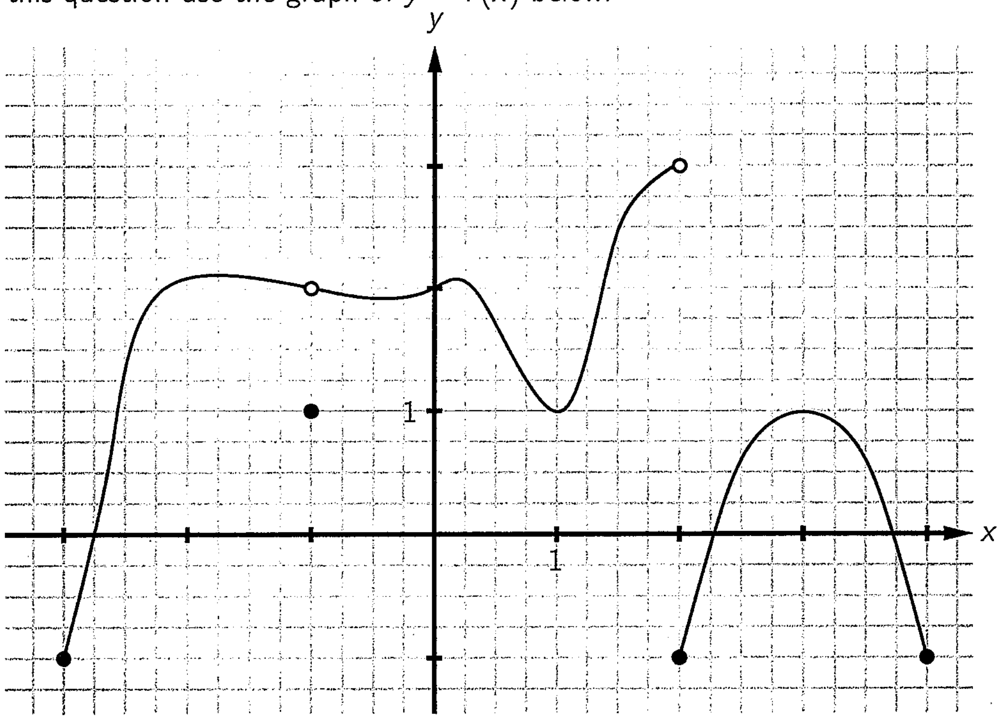
Question 1: For this question use the graph of $y = f(x)$ below:



(a)[2] What is $(f \circ f)(2)$?

$f(f(2)) = f(-1) = \boxed{1}$

(b)[1] State the domain of $f(x)$ using interval notation.

$[-3, 4]$

(c)[1] State the range of $f(x)$ using interval notation.

$[-1, 3)$

(d)[2] Determine $\lim_{x \to -1} f(x)$.

$\boxed{\lim_{x \to -1} f(x) = 2}$

(e)[2] Determine $\lim_{x \to 2} f(x)$.

$\lim_{x \to 2^-} f(x) = 3$; $\lim_{x \to 2^+} f(x) = -1$)

So $\lim_{x \to 2} f(x)$ $\boxed{\text{does not exist}}$

(f)[2] What is $\lim_{x \to 2^-} f(x)$?

$\lim_{x \to 2^-} f(x) = \boxed{3}$

Question 2: For this question use the functions $f(x) = \dfrac{1}{x-1}$ and $g(x) = \sqrt{x-3}$.

(a)[3] Determine $(f-g)(x)$ and state the domain using interval notation.

$$(f-g)(x) = \frac{1}{x-1} - \sqrt{x-3}.$$

For domain, must have $x-3 \geq 0$, $x \neq 1$

$\therefore$ $x \geq 3$, $x \neq 1$

$\therefore$ $\boxed{[3, \infty)}$

(b)[3] Determine $(f \circ g)(x)$ and state the domain using interval notation.

$$(f \circ g)(x) = \frac{1}{\sqrt{x-3}-1}.$$

For domain, must have $x-3 \geq 0$, $\sqrt{x-3} \neq 1$

$\Rightarrow$ $x \geq 3$, $x \neq 4$

$\therefore$ $\boxed{[3,4) \cup (4,\infty)}$

(c)[4] Compute and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[\frac{1}{x+h-1} - \frac{1}{x-1}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{x}\cancel{-1} - \cancel{x} - h \cancel{+1}}{(x+h-1)(x-1)}\right]$$

$$= \boxed{\frac{-1}{(x+h-1)(x-1)}}$$

Question 3: Evaluate the following limits, if they exist:

(a)[3] $\lim_{x\to 3} \frac{x^2+4x-21}{x^2-x-6}$ $\sim$ "$\frac{0}{0}$"

$$= \lim_{x\to 3} \frac{(x+7)(x-3)}{(x+2)(x-3)}$$

$$= \frac{10}{5}$$

$$= \boxed{2}$$

(b)[3] $\lim_{x\to 5^-} \frac{x-\sqrt{5-x}}{5+x}$ $= \frac{5-0}{10} = \boxed{\frac{1}{2}}$

(c)[4] $\lim_{x\to 2^-} \left(\frac{1}{x-2}+\frac{1}{|x-2|}\right)$ $= \lim_{x\to 2^-} \left(\frac{1}{x-2}+\frac{1}{-(x-2)}\right)$

$$= \lim_{x\to 2^-} 0$$

$$= \boxed{0}$$

Question 4: Evaluate the following limits, if they exist:

(a)[5] $\lim_{x\to 4} \frac{4-\sqrt{12+x}}{x-4}$ $\sim$ "$\frac{0}{0}$"

$$= \lim_{x\to 4} \frac{4-\sqrt{12+x}}{x-4} \cdot \frac{4+\sqrt{12+x}}{4+\sqrt{12+x}}$$

$$= \lim_{x\to 4} \frac{16-12-x}{(x-4)(4+\sqrt{12+x})}$$

$$= \lim_{x\to 4} \frac{(4-x)}{(x-4)(4+\sqrt{12+x})}$$

$$= \lim_{x\to 4} \frac{-\cancel{(x-4)}}{\cancel{(x-4)}(4+\sqrt{12+x})} = \boxed{\frac{-1}{8}}$$

(b)[5] $\lim_{x\to -6} \frac{\left[\frac{1}{6}+\frac{1}{x}\right]}{6+x}$ $\sim$ "$\frac{0}{0}$"

$$= \lim_{x\to -6} \frac{1}{\cancel{6+x}}\left[\frac{\cancel{x+6}}{6x}\right] = \boxed{\frac{-1}{36}}$$

Question 5:

(a)[3] Evaluate the following limit if it exists: $\lim_{x \to 0} \dfrac{\sin(2x)}{\sin(7x)\cos(3x)}$ $\sim$ "$\frac{0}{0}$"

$$\lim_{x \to 0} \frac{\sin(2x)}{\sin(7x)\cos(3x)} = \lim_{x \to 0} \left(\frac{\sin(2x)}{2x}\right) \cdot \frac{1}{\left(\frac{\sin(7x)}{7x}\right)} \cdot \frac{2x}{7x \cdot \cos(3x)}$$

$$= \boxed{\frac{2}{7}}$$

(b)[3] Evaluate the following limit if it exists: $\lim_{\theta \to 0} \dfrac{\sin(2\theta)}{4\theta^2}$

$$\lim_{\theta \to 0} \frac{\sin(2\theta)}{4\theta^2} = \lim_{\theta \to 0} \left(\frac{\sin(2\theta)}{2\theta}\right) \cdot \frac{1}{2\theta} \sim \text{"}\frac{1}{0}\text{"}$$

Limit does not exist.

(~~b~~ c)[4] Suppose $g(x)$ is a function with the property that $-2 \le g(x) \le 3$ for every real number x. Determine $\lim_{x \to 0} x^4 g(x)$. (State any theorems used, like the the Squeeze Theorem, for example, and be sure to state the conditions necessary to justify use of the theorem.)

$-2 \le g(x) \le 3$, so $-2x^4 \le x^4 g(x) \le 3x^4$

since $\lim_{x \to 0} -2x^4 = 0 = \lim_{x \to 0} 3x^4$,

by the Squeeze Theorem, $\lim_{x \to 0} x^4 g(x) = 0$.