Question 1:

(a)[3] Over a 9 month period an initial investment of \$23,500 earned \$1057.50 of simple interest. What was the annual rate of simple interest? (State answer as a percentage rounded to 1 decimal place.)

$t = \frac{9}{12}$

$P = 23{,}500$

$I = 1057.50$

$I = Prt$

$r = \frac{I}{Pt}$

$= \frac{1057.50}{(23500)(\frac{9}{12})}$

$= \boxed{6.0\%}$

(b)[3] Determine the rate of interest compounded semiannually which is equivalent to an effective rate of 7.25%. (State answer as a percentage rounded to 1 decimal place.)

$\left(1 + \frac{r}{n}\right)^n = 1 + R$ where $R = 0.0725$, $n = 2$

$\therefore \quad 1 + \frac{r}{n} = (1+R)^{\frac{1}{n}}$

$r = n\left[(1+R)^{\frac{1}{n}} - 1\right] = 2\left[(1+0.0725)^{\frac{1}{2}} - 1\right]$

$\approx \boxed{7.1\%}$

(c)[4] An amount P is invested at 5% compounded quarterly. An amount of \$300 is invested separately at 4% compounded continuously. The total value of both investments after two years is \$1000. Determine the value of P. (Round final answer to 2 decimal places.)

Investment 1: $r = 0.05$, $n = 4$, $t = 2$

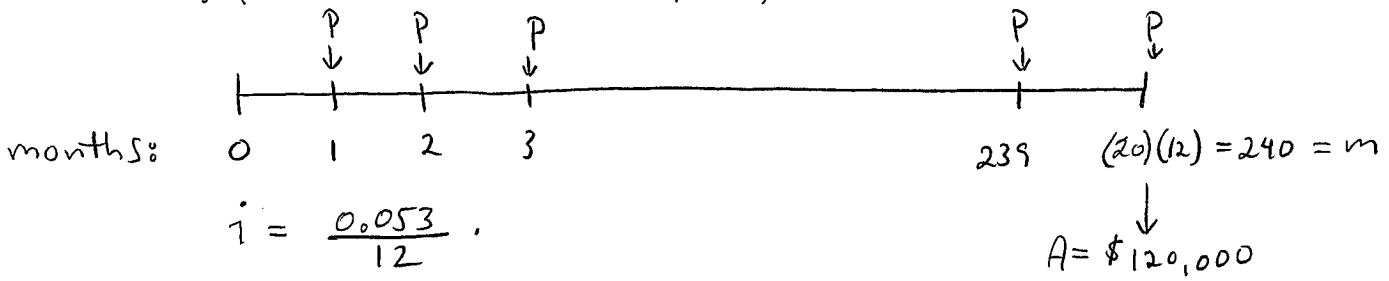
Investment 2: $P = 300$, $r = 0.04$ (continuous), $t = 2$.

$\therefore \quad P\left(1 + \frac{0.05}{4}\right)^{(4)(2)} + 300e^{(0.04)(2)} = 1000$

$\therefore \quad P = \frac{1000 - 300e^{0.08}}{\left(1 + \frac{0.05}{4}\right)^8} \approx \boxed{\$611.16}$

Question 2:

(a)[5] Determine the payments that must be made to an investment at the end of each month for the next 20 years in order to accumulate \$120,000. The investment earns interest at a rate of 5.3% compounded monthly. (Round final answer to 2 decimal places.)

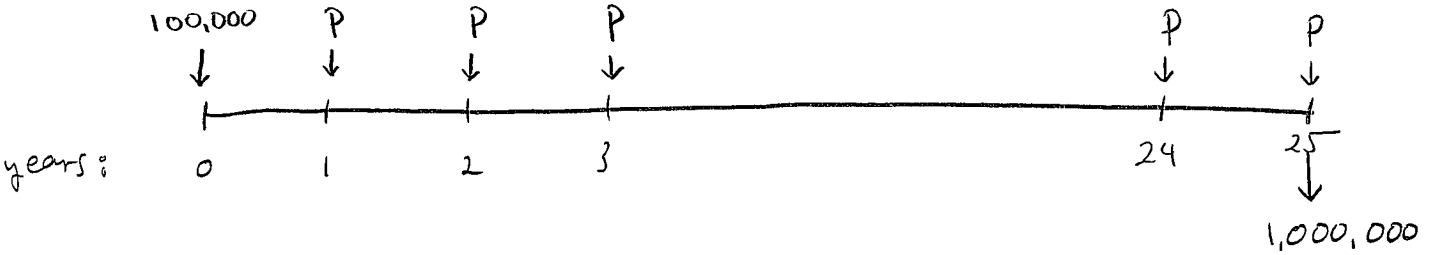


$i = \frac{0.053}{12}$.

$$A = P\left[\frac{(1+i)^m - 1}{i}\right]$$

$$\therefore P = \frac{A}{\left[\frac{(1+i)^m - 1}{i}\right]} = \frac{120{,}000}{\left[\frac{\left(1+\frac{0.053}{12}\right)^{240} - 1}{\left(\frac{0.053}{12}\right)}\right]} \approx \boxed{\$281.97}$$

(b)[5] On his 40th birthday Mr. Smithers decides to get serious about his retirement plan. His goal is to have \$1,000,000 saved by his 65th birthday. He currently has \$100,000 in a fund which earns interest at 6% compounded annually. How much should he contribute to the fund at the end of each year to reach his goal? (Round final answer to 2 decimal places.)



A_1 = Value of 100,000 at end of 25 years.

$$= 100{,}000\,(1+0.06)^{25}$$

A_2 = value of annuity at end of 25 years.

$$= P\left[\frac{(1+0.06)^{25} - 1}{0.06}\right]$$

Need $100{,}000\,(1.06)^{25} + P\left[\frac{(1.06)^{25} - 1}{0.06}\right] = 1{,}000{,}000$

$$\therefore P = \frac{1{,}000{,}000 - 100{,}000\,(1.06)^{25}}{\left[\frac{(1.06)^{25} - 1}{0.06}\right]} \approx \boxed{\$10{,}404.05}$$

Question 3:

(a)[5] A credit card charges 19% interest compounded monthly. Suppose you use the credit card to make a purchase today, and in order to pay off the credit card you will be required to make payments of \$122 at the end of each month for the next three years. What was the cost of the purchase you made? (Round final answer to 2 decimal places.)

$i = \frac{0.19}{12}$.

$$V = P\left[\frac{1-(1+i)^{-m}}{i}\right]$$

$$= 122\left[\frac{1-(1+\frac{0.19}{12})^{-36}}{(\frac{0.19}{12})}\right] \approx \boxed{\$3328.24}$$

(b)[5] A 30 year old plans to retire at age 60. He plans to make deposits at the end of each month for the next 30 years into an account paying 4.6% compounded monthly so that when he retires he can withdraw \$300 at the end of each month for 35 years. How much should the monthly deposits be? (Round final answer to 2 decimal places.)

Let A_1 = funds required at age 65 to pay pension
= present value of \$300 per month for 35 years at 4.6% compounded monthly.

$\therefore i = \frac{0.046}{12}$, $m = (35)(12) = 420$, $P_1 = 300$

$$\therefore A_1 = P_1\left[\frac{1-(1+i)^{-m}}{i}\right] = 300\left[\frac{1-(1+\frac{0.046}{12})^{-420}}{(\frac{0.046}{12})}\right] \approx \$62,569.27$$

Let P_2 = monthly payments required to save up A_1 over next 30 years.

This time $i = \frac{0.046}{12}$, $m = (30)(12) = 360$, so

$$A_1 = P_2\left[\frac{(1+i)^m - 1}{i}\right] \Rightarrow P_2 = \frac{62,569.27}{\left[\frac{(1+\frac{0.046}{12})^{360}-1}{(0.046/12)}\right]} \approx \boxed{\$80.91}$$

Question 4: For this question use the following sets:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \quad A = \{2, 4, 6, 8\}, \quad B = \{2, 3, 4, 5, 6\}, \quad C = \{1, 2, 3, 8, 9\}$$

Determine the following:

(a)[2] $A \cap B$ $= \{2, 4, 6\}$

(b)[2] $\overline{A} \cap C$ $= \{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 8, 9\}$

$= \{1, 3, 9\}$

(c)[2] $B \cap (A \cup C)$ $= B \cap \{1, 2, 3, 4, 6, 8, 9\}$

$= \{2, 3, 4, 6\}$

(d)[2] $B \cup \overline{(A \cup C)}$ $= B \cup \overline{\{1, 2, 3, 4, 6, 8, 9\}}$

$= B \cup \{5, 7\}$

$= \{2, 3, 4, 5, 6, 7\}$

(e)[2] $n(A \cup B) - n(A \cap B)$

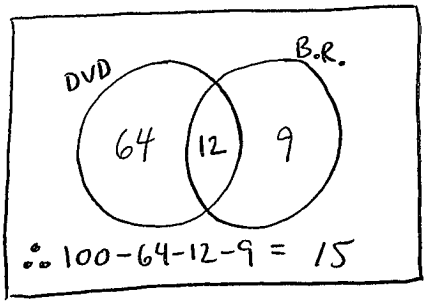
$= n(\{2, 3, 4, 5, 6, 8\}) - n(\{2, 4, 6\})$

$= 6 - 3$

$= 3$

Question 5:

(a)[3] A survey of 100 households finds that 76 have a DVD player, 21 have a Blue Ray player, while 12 households have both. How many households have neither machine?



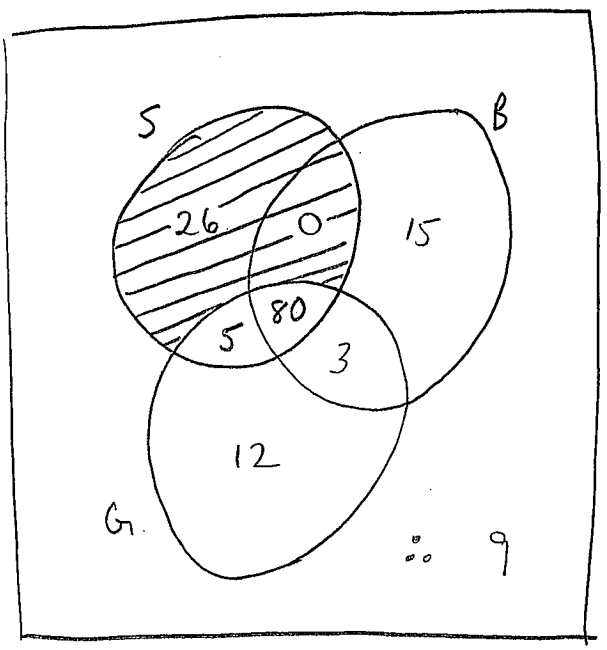
15 have neither machine.

(b)[7] A survey of 150 investors resulted in the following data:

111 invested in stocks; ✓
98 invested in bonds; ✓
100 invested in GICs; ✓
80 invested in stocks and bonds; ✓
83 invested in bonds and GICs; ✓
85 invested in stocks and GICs; ✓
80 invested in all three investments ✓

How many investors invested in stocks but not GICs?

S: Stocks, B: bonds ; G: GICs.



▨ : S but not G.

∴ 26 invested in stocks but not GICs.