Question 1:

(a)[2] Determine the slope of the line through the points $(-1, 4)$ and $(1, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{1-(-1)} = \frac{2}{2} = \boxed{1}$$

(b)[2] Determine an equation of the line having slope -7 and y-intercept $(0, -3)$.

$$y = mx + b$$

$$\boxed{y = -7x - 3}$$

(c)[2] Determine an equation of the vertical line through the point $\left(\frac{1}{2}, 3\right)$.

$$\boxed{x = \frac{1}{2}}$$

(d)[2] Determine the x-intercept of the line $y - 6 = -2(x + 3)$.

$$y - 6 = -2(x+3)$$

$$0 - 6 = -2(x+3)$$

$$3 = x + 3$$

$$x = 0$$

$$\boxed{\therefore (0, 0)}$$

(e)[2] Determine the slope of the line $4x + 3y = 24$.

$$4x + 3y = 24$$

$$3y = -4x + 24$$

$$y = -\frac{4}{3}x + 8$$

$$\boxed{\therefore m = -\frac{4}{3}}$$

Question 2:

(a)[4] Determine whether the following lines are intersecting, parallel or coincident:

$$L: -3x + 12y = 4$$
$$M: \quad 6x + 8y = 1$$

L: $12y = 3x + 4$

$y = \frac{3}{12}x + \frac{4}{12} = \frac{1}{4}x + \frac{1}{3}$

M: $8y = -6x + 1$

$y = \frac{-6}{8}x + \frac{1}{8} = -\frac{3}{4}x + \frac{1}{8}$

Slopes differ, so lines are intersecting.

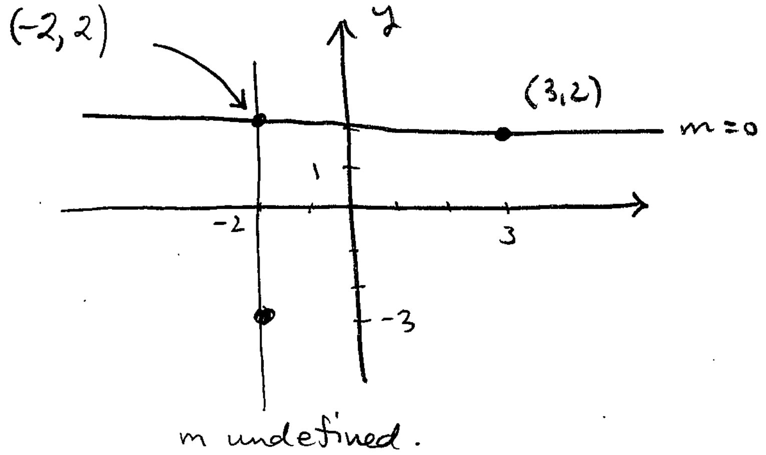
(b)[3] Determine an equation of the line through the point $(1, 1)$ which is parallel to the line with x-intercept $(-2, 0)$ and y-intercept $(0, 1)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0-(-2)} = \frac{1}{2}$

$\therefore \quad y - y_0 = m(x - x_0)$

$\boxed{y - 1 = \frac{1}{2}(x-1)}$ or $\boxed{y = \frac{1}{2}x + \frac{1}{2}}$

(c)[3] Determine the point of intersection of the line of slope zero through $(3, 2)$ and the line with slope undefined through $(-2, -3)$.



Point of intersection is $(-2, 2)$.

Question 3:

(a)[5] \$10,000 is invested in a mix of two investments. One investment pays interests at an annual rate of 8%, while the second pays 12%. If \$1000 in interest is earned during the first year, how much was invested in each of the investments? Clearly define your variables and state a clear conclusion.

Let x = amount invested at 8%,
y = amount invested at 12%.

① $x + y = 10{,}000$

② $0.08x + 0.12y = 1000$

① $\Rightarrow$ $y = 10{,}000 - x$

② $\Rightarrow$ $0.08x + 0.12(10\,000 - x) = 1000$

$0.08x + 1200 - 0.12x = 1000$

$-0.04x = -200$

$x = 5000$

$\therefore$ $y = 10\,000 - 5000 = 5000$

$\boxed{\therefore \text{\$5000 is invested at 8\%, \$5000 is invested at 12\%}}$

(b)[5] A store sells cashews for \$5.00 per pound and peanuts for \$1.50 per pound. 30 pounds of peanuts will be mixed with some cashews and the mixture will sell for \$3.00 per pound. How many pounds of cashews should be mixed with the peanuts so that the sale of the mixture will produce the same revenue as would selling the ingredient nuts separately?

Let x = number of pounds of cashews.

$$\underbrace{(30+x)(3)}_{\text{revenue from mixture}} = \underbrace{(30)(1.50) + (x)(5)}_{\text{revenue if ingredient nuts sold separately.}}$$

So $90 + 3x = 45 + 5x$

$2x = 45$

$x = 22.5$

$\boxed{\therefore \text{22.5 pounds of cashews should be used.}}$

Question 4:

(a)[5] A doughnut shop sells doughnuts for \$3.29 per dozen. The shop has fixed weekly costs of \$650 and it costs \$1.55 to make a dozen doughnuts. How many dozen doughnuts must be sold each week in order for the business to break even? Round your final answer to the nearest dozen.

Let x = number of dozen sold.

$R = 3.29x$

$C = 650 + 1.55x.$

$$R = C \implies 3.29x = 650 + 1.55x$$
$$1.74x = 650$$
$$x = \frac{650}{1.74} \approx 374 \text{ dozen.}$$

$\therefore$ Approximately 374 dozen must be sold each week.

(b)[5] Solve the following system of equation using any method you wish. Based on your solution, state whether the system is consistent or inconsistent.

$$\text{①} \quad 3x - 6y = 2$$
$$\text{②} \quad 5x + 4y = 1$$

$$\text{①} \Rightarrow 6y = 3x - 2$$
$$y = \tfrac{1}{2}x - \tfrac{1}{3}$$

$$\text{②} \Rightarrow 5x + 4\left(\tfrac{1}{2}x - \tfrac{1}{3}\right) = 1$$
$$5x + 2x - \tfrac{4}{3} = 1$$
$$7x = \tfrac{7}{3}$$
$$x = \tfrac{1}{3}$$

$$\therefore y = \tfrac{1}{2}\left(\tfrac{1}{3}\right) - \tfrac{1}{3} = \tfrac{1}{6} - \tfrac{2}{6} = -\tfrac{1}{6}$$

$\therefore$ solution is $\left(\frac{1}{3}, -\frac{1}{6}\right)$; system is <u>consistent</u>

Question 5:

(a)[5] A company has data indicating that when the price of a particular product is \$138 the quantity demanded is 72 while the quantity supplied is 96. At the market (or equilibrium) price of \$120 the quantity demanded increases to 88. Determine the supply equation for the product.

(138, 72) is on D-line.
(138, 96) is on S-line.
(120, 88) is on D-line and S-line since $p = \$120$ is market price.

$$m_s = \frac{96-88}{138-120} = \frac{8}{18} = \frac{4}{9}$$

$$\therefore \quad S - S_1 = m_s(p - p_1)$$

$$\boxed{S - 96 = \frac{4}{9}(p - 138)} \quad \text{or} \quad \boxed{S = \frac{4}{9}p + \frac{104}{3}}$$

(b)[5] In 1990, 322 out of every 100,000 people in the United States died of heart disease. By 2004 that number had dropped to 217 out of every 100,000 people. Assuming a linear relationship between time (in years) and deaths due to heart disease, predict the number of deaths per 100,000 people that would be expected in the year 2010.

t	y
1990	322
2004	217

$$m = \frac{322-217}{1990-2004} = \frac{-105}{14} = \frac{-15}{2}$$

$$\therefore \quad y - y_1 = m(t - t_1)$$

$$y - 217 = \frac{-15}{2}(t - 2004).$$

When $t = 2010$:
$$y - 217 = \frac{-15}{2}(2010 - 2004)$$
$$y - 217 = \frac{-15}{2}(6)$$
$$y = -45 + 217 = 172$$

$\boxed{\therefore \text{ 172 deaths per 100,000 would be expected.}}$