 Name: SOLUTIONS.

(1) [7] Let

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & -2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 3 & 1 \\ 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

Compute $3\mathbf{CB} + 4\mathbf{D}$.

$$3CB + 4D = 3 \begin{bmatrix} 3 & 1 \\ 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & -2 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 2 & 10 & 7 \\ 5 & 4 & 14 \\ -2 & 8 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 16 \\ 0 & 4 & 8 \\ 0 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 30 & 21 \\ 15 & 12 & 42 \\ -6 & 24 & -12 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 16 \\ 0 & 4 & 8 \\ 0 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 30 & 37 \\ 15 & 16 & 50 \\ -6 & 20 & -8 \end{bmatrix}$$

(2) [8] Let $\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Determine $\mathbf{A}^{-1}$. Clearly label all row operations used.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = (-2)r_1 + r_2$;
$R_3 = (-1)r_1 + r_3$;

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & -2 & 1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$R_2 = (-1)r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$R_1 = (-1)r_2 + r_1$;
$R_3 = r_2 + r_3$;

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 \\ 0 & 1 & -3 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right]$$

$R_3 = (-1)r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 \\ 0 & 1 & -3 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

$R_1 = (-2)r_3 + r_1$:
$R_2 = (3)r_3 + r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & -1 & 2 & -3 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ -1 & 1 & -1 \end{bmatrix}$$