(1) [15] Use matrix reduction to solve the following system of equations:

$$\begin{aligned} 2x + y - z &= 2 \\ x + 3y + 2z &= 1 \\ x + y + z &= 2 \end{aligned}$$

You may use either Gaussian elimination or Gauss-Jordan elimination. Clearly state the row operations used at each step of the reduction, and state a clear conclusion.

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{array}\right]$$

$r_1 \leftrightarrow r_3$:

$$\left[\begin{array}{ccc|c} \textcircled{1} & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 2 & 1 & -1 & 2 \end{array}\right]$$

$R_2 = (-1)r_1 + r_2$:
$R_3 = (-2)r_1 + r_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & -1 & -3 & -2 \end{array}\right]$$

$r_2 \leftrightarrow r_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -2 \\ 0 & 2 & 1 & -1 \end{array}\right]$$

$R_2 = (-1)r_2$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & \textcircled{1} & 3 & 2 \\ 0 & 2 & 1 & -1 \end{array}\right]$$

$R_1 = (-1)r_2 + r_1$:
$R_3 = (-2)r_2 + r_3$:

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & -5 & -5 \end{array}\right]$$

$R_3 = \frac{(-1)}{5} r_3$:

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & \textcircled{1} & 1 \end{array}\right]$$

$R_1 = (2)r_3 + r_1$:
$R_2 = (-3)r_3 + r_2$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore x = 2,\ y = -1,\ z = 1$