VANCOUVER ISLAND
UNIVERSITY

Math 251 Test 2 – Apr 27 2011

name (printed)	*student number*

Instructions:

1. **DO 5 of 6 QUESTIONS**
2. There are **8 pages** (including this cover page) in the test. You will be given **75 minutes** to write the test. Justify every answer, and clearly show your work. Unsupported answers will receive no credit. Read over the test before you begin.
3. You may use a single letter-size "cheat sheet" containing formulae, theory and numerical values, however **your cheat sheet may not contain worked examples.** The instructor will have the final decision on what is or is not appropriate for the cheat sheet. Hand in your cheat sheet along with your completed test. **To be considered for grading, your test must include your cheat sheet.**
4. A table of formulas for the Method of Undetermined Coefficients is included with the test.
5. Other than the cheat sheet and formula sheet noted above, no notes or books are to be used during the test. The last page is for scrap work. Put your name on the scrap paper and return it along with your completed test.
6. A basic scientific non-programmable, non-graphing calculator is permitted, however calculators may not be shared.
7. At the end of the test you will be given the instruction to stop writing. **Continuing to write after this instruction is cheating**.
8. **Academic dishonesty:** Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the test, a zero grade in the course, and other measures, such as suspension from this university.

question	value	score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Question 1:

(a)[6] Given that $y_1(x) = x$ is one solution of

$$x^2y'' + 2xy' - 2y = 0, \qquad x > 0,$$

use reduction of order to find a second linearly independent solution $y_2(x)$. (Do not use "the formula", but rather, find $y_2(x)$ from first principles. You may check your result using the formula however.)

(b)[4] Compute the Wronskian $W(y_1, y_2)$ of the solution functions from part (a). What property of the Wronskian confirms that y_1 and y_2 are linearly independent on the interval $(0, \infty)$?

Question 2 [10]: Solve the following initial value problem:

$$y'' + 4y' + 5y = 0, \qquad y(0) = 1, \quad y'(0) = 0$$

Question 3 [10]: Determine the general solution:

$$y'' + 6y' + 5y = e^{-x}$$

Question 4 [10]: The temperature $u(r)$ between concentric circles of radius $r = a$ and $r = b$, (where $a < b$) is determined by the boundary value problem

$$r\frac{d^2u}{dr^2} + \frac{du}{dr} = 0, \qquad u(a) = u_0, \quad u(b) = u_1,$$

where u_0 and u_1 are constants. Solve for $u(r)$.

Question 5 [10]: A 1 kg mass is attached to a spring whose constant is $k = 16$ N/m. The entire spring-mass system is then submerged in a liquid that imparts a damping force of $\beta = 10$ times the velocity of the mass. At time $t = 0$ the mass is released from a point 1 m below equilibrium with an initial velocity of 12 m/s upward. Set up and solve the differential equation describing the position $x(t)$ of the mass for $t \geq 0$.

Question 6 [10]: Determine the general solution $\mathbf{X}(t)$ of the following system. Express your final answer in a form that does not contain complex numbers.

$$\mathbf{X}'(t) = \begin{bmatrix} 5 & 1 \\ -2 & 3 \end{bmatrix} \mathbf{X}(t)$$