VANCOUVER ISLAND
UNIVERSITY

Math 251 Test 1 – Feb 9 2011

name (printed)	*student number*

Instructions:

1. There are **7 pages** (including this cover page) in the test. You will be given **75 minutes** to write the test. Justify every answer, and clearly show your work. Unsupported answers will receive no credit. Read over the test before you begin.

2. You may use a single letter-size "cheat sheet" containing formulae, theory and numerical values, however **your cheat sheet may not contain worked examples.** The instructor will have the final decision on what is or is not appropriate for the cheat sheet. Hand in your cheat sheet along with your completed test. **To be considered for grading, your test must include your cheat sheet.**

3. Other than the cheat sheet noted above, no notes or books are to be used during the test. The last page is for scrap work. Put your name on the scrap paper and return it along with your completed test.

4. A basic scientific non-programmable, non-graphing calculator is permitted, however calculators may not be shared.

5. At the end of the test you will be given the instruction to stop writing. **Continuing to write after this instruction is cheating**.

6. **Academic dishonesty:** Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the test, a zero grade in the course, and other measures, such as suspension from this university.

question	value	score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Question 1:

(a)[5] Solve the following IVP:

$$\frac{dy}{dx} = \frac{y \cos x}{1 + 2y^2}, \qquad y(\pi/2) = 1$$

(b)[5] Solve the following IVP:

$$y' + 2y = xe^{-2x}, \qquad y(1) = 0$$

Question 2:

(a)[5] Solve the following differential equation:

$$(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)\frac{dy}{dx} = 0$$

(b)[5] Find a non-trivial solution to the following differential equation:

$$x^2\frac{dy}{dx} - 2xy = 3y^4$$

Question 3: For this question consider the differential equation

$$\frac{dy}{dx} = y \ln(1 + y^2) - y \ln 5$$

(a)[2] Determine the equilibrium solution(s).

(b)[2] Sketch the one-dimensional phase portrait.

(c)[2] Classify each critical point.

(d)[2] Graph the equilibrium solutions and sketch typical solution curves in the regions between the equilibria.

(e)[2] Suppose we are given the initial condition $y(100) = \frac{1}{e}$. What is $\lim_{x \to \infty} y(x)$?

Question 4: A tank contains 50 kg of salt dissolved in 1000 L of water. Water with a concentration of 0.1 kg/L begins entering the tank at a rate of 2 L/s, while the well-mixed solution is pumped out the bottom of the tank at the same rate.

(a)[2] Write the IVP which models the amount $A(t)$ (in kg) of dissolved salt in the tank at time t.

(b)[6] Determine how much dissolved salt is in the tank after five minutes.

(c)[2] Is there a time t at which the amount of salt in the tank will be double its initial value? Explain.

Question 5: For this question use the IVP $y' = 0.5 - x + 2y, \qquad y(0) = 1$.

(a)[4] Use Euler's Method with step size $h = 0.1$ to approximate $y(0.2)$ to three decimals.

(b)[4] Solve the IVP to determine the actual solution.

(c)[2] Determine the relative error in your approximation (a).

(d)[2 bonus marks] Suppose you don't know the actual solution of the IVP. Can you think of a way of using the IVP itself to determine if your approximation in (a) is an over or under-estimate?

SCRAP - Test 1 **Name:**