VANCOUVER ISLAND
UNIVERSITY

Math 251 Practice Test 1 – Feb 9 2011

name (printed)	*student number*

Instructions:

1. There are **8 pages** (including this cover page) in the test. You will be given **75 minutes** to write the test. Justify every answer, and clearly show your work. Unsupported answers will receive no credit. Read over the test before you begin.
2. You may use a single letter-size "cheat sheet" containing formulae, theory and numerical values, however **your cheat sheet may not contain worked examples.** The instructor will have the final decision on what is or is not appropriate for the cheat sheet. Hand in your cheat sheet along with your completed test. **To be considered for grading, your test must include your cheat sheet.**
3. Other than the cheat sheet noted above, no notes or books are to be used during the test. The last page is for scrap work. Put your name on the scrap paper and return it along with your completed test.
4. A basic scientific non-programmable, non-graphing calculator is permitted, however calculators may not be shared.
5. At the end of the test you will be given the instruction to stop writing. **Continuing to write after this instruction is cheating**.
6. **Academic dishonesty:** Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the test, a zero grade in the course, and other measures, such as suspension from this university.

question	value	score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

Question 1:

(a)[5] Solve the following differential equation and determine the largest interval I on which the solution is defined:

$$\frac{dy}{dt} + 2(t+1)y^2 = 0, \qquad y(0) = -1/8$$

(b)[5] Solve the following differential equation:

$$\frac{dx}{dy} = -\frac{4y^2 + 6xy}{3y^2 + 2x}$$

Question 2:

(a)[5] Solve the following differential equation:

$$(x+2)^2 \frac{dy}{dx} = 5 - 8y - 4xy$$

(b)[5] Solve the following differential equation:

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

Question 3:

(a)[5] Solve the following differential equation:

$$\frac{dy}{dx} - y = e^x y^2$$

(b)[5] Solve the following differential equation:

$$\frac{dy}{dx} = \cos(x + y), \qquad y(0) = \pi/4$$

Question 4: For this question consider the differential equation

$$\frac{dP}{dt} = aP \ln\left(\frac{b}{P}\right)$$

where a and b are positive constants.

(a)[4] Determine the equilibrium solution(s).

(b)[5] Solve the differential equation.

(c)[1] What is $\lim_{t\to\infty} P(t)$?

Question 5:

(a)[5] A fish population is decreasing at a rate that is proportional to the square root of its size. Initially there were 90,000 fish, and after six weeks 40,000 remain. Solve the corresponding differential equation to determine the time at which the population will be reduced to 10,000.

(b)[5] Consider the autonomous differential equation

$$\frac{dy}{dx} = \frac{ye^y - 9y}{e^y}$$

(i) Determine the critical points and sketch the one-dimensional phase portrait.

(ii) Classify each critical point as asymptotically stable, unstable, or semi-stable.

(iii) Graph the equilibrium solutions and sketch typical solution curves in the regions between the equilibria.

Question 6:

(a)[4] For this question use the IVP $y' = 2y\cos(x), \quad y(0) = 1$.

(i) Use Euler's Method with step size $h = 0.1$ to approximate $y(0.2)$.

(ii) The actual solution to the IVP is $y = e^{2\sin x}$. Use this fact to determine the relative error in your approximation in part (i).

(b)[6] Write a differential equation which models each of the following situations, and state a reasonable initial condition for each. Do not solve the differential equations.

(i) The acceleration of a car is proportional to the difference between 250 km/h and the velocity of the car.

(ii) The size of an alligator population in a region follows a logistic model. Hunting is then allowed and alligators are removed from the region at a rate proportional to the existing population size.

(iii) In the theory of learning, the rate at which a subject is learned is assumed to be proportional to the amount that is left to be learned. Let $A(t)$ represent the amount learned by time t, and M be the total amount to be learned.