Question 1:

(a)[5] Determine $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(b)[5] Determine $\displaystyle\int \sin^3 x\,\cos^2 x\,dx$

Question 2:

(a)[5] Compute $\int_1^2 \frac{\ln x}{x^2}\,dx$

(b)[5] Determine $\int \frac{1}{x(x-1)^2}\,dx$

Question 3:

(a)[5] A sphere of radius r has volume $V = 4\pi r^3/3$. Derive this formula using integration.

(b)[2] The region bounded by $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/4$ is rotated about the x-axis. Set up but do not evaluate the integral representing the volume of the resulting solid.

(b)[3] The region bounded by $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/4$ is rotated about the line $y = 2$. Set up but do not evaluate the integral representing the volume of the resulting solid.

Question 4:

(a)[5] Evaluate the following improper integral where $p > 1$ is a constant. Show all steps including any required limits.

$$\int_e^\infty \frac{1}{x(\ln x)^p}\,dx$$

(b)[5] Solve the following differential equation:

$$(1+\cos x)y' = e^{-y}\sin x, \qquad y(0) = 0$$

You may leave your solution in implicit form (it is not necessary to isolate y in your final answer.)

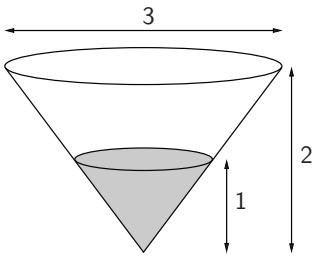
Question 5:

(a)[5 points] Use T_4, the trapezoid rule on four subintervals, to approximate the integral $\int_0^{\pi} \sin^2 x \, dx$.

(b)[5 points] Determine the error in your approximation T_4 in part (a). Recall, the error in using the trapezoid rule to approximate $\int_a^b f(x) \, dx$ is at most $\frac{K(b-a)^3}{12n^2}$, where $|f''(x)| \leq K$ on $[a, b]$.

Question 6:

(a)[5] The cone shaped vessel shown below has been filled with water to a depth of 1 m. Determine the amount of work required to empty the cone by pumping water up and over the top rim. Recall that the density of water is $\rho = 1000$ kg/m^3 and acceleration due to gravity is $g = 9.8$ m/s^2. You may leave the constants ρ and g in your final answer if you like. State units with your answer.



(b)[5] A 20 m rope of mass 4 kg hangs over the side of a building. How much work is done in pulling half of the rope up onto the roof? Recall that acceleration due to gravity is $g = 9.8$ m/s^2. You may leave the constant g in your final answer if you like. State units with your answer.