Question 1:

(a)[3] Determine $\lim_{x\to0^+} \arctan(-1/\sqrt{x})$.

As $x \to 0^+$, $\frac{-1}{\sqrt{x}} \to -\infty$,

so $\arctan\left(\frac{-1}{\sqrt{x}}\right) \to -\frac{\pi}{2}$

$\therefore \lim_{x\to0^+} \arctan\left(\frac{-1}{\sqrt{x}}\right) = \boxed{-\frac{\pi}{2}}$

(b)[4] Determine an equation of the tangent line to the curve $y = 3\arccos(x/2)$ at the point where $x = 1$.

At $x=1$, $y = 3\arccos\left(\frac{1}{2}\right) = 3\left(\frac{\pi}{3}\right) = \pi$

$\therefore (1,\pi)$ is a point on the line.

Also, slope of line is $\left.\frac{dy}{dx}\right|_{x=1} = \left.\frac{-3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}\right|_{x=1}$

$= -\sqrt{3}$

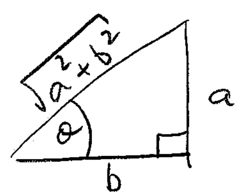
$\therefore$ Equation of tangent line is

$\boxed{y - \pi = -\sqrt{3}(x-1)}$

(c)[3] Simplify sec(arctan(a/b)). Your final answer should not contain any trigonometric or inverse trigonometric functions.

Let $\theta = \arctan\left(\frac{a}{b}\right)$

$\therefore \tan\theta = \frac{a}{b}$



$\therefore \sec\left(\arctan\left(\frac{a}{b}\right)\right) = \sec(\theta) = \boxed{\frac{\sqrt{a^2+b^2}}{b}}$

Question 2:

(a)[6] Determine the two values of x at which the tangent lines to the curve $y = \sinh x$ have slope 2 .

Solve $y' = 2$: $y' = \cosh x$

$$\cosh x = 2$$

$$\frac{e^x + e^{-x}}{2} = 2$$

$$e^x + e^{-x} = 4$$

$$e^{2x} + 1 = 4e^x$$

$$(e^x)^2 - 4(e^x) + 1 = 0$$

$$\therefore e^x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

$$\boxed{\therefore x = \ln(2+\sqrt{3}), \ln(2-\sqrt{3})}$$

(b)[4] Let $f(x) = \sinh(x + \sinh^2 x)$. Determine $f'(0)$.

$$f'(x) = \cosh(x + \sinh^2 x) \cdot (1 + 2\sinh x \cosh x)$$

$$f'(0) = \cosh(0 + \underbrace{\sinh^2(0)}_{0})(1 + 2\underbrace{\sinh(0)}_{0}\cosh(0))$$

$$= \boxed{1}$$

Question 3:

(a)[4] Evaluate $\lim_{x\to-\infty} x^2e^{2x}$. $\sim$ "$\infty \cdot 0$" } indeterminate

$$\lim_{x\to-\infty} x^2 e^{2x} = \lim_{x\to-\infty} \frac{x^2}{e^{-2x}} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to-\infty} \frac{2x}{-2e^{-2x}} \sim \text{"}\frac{-\infty}{-\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to-\infty} \frac{2}{4e^{-2x}}$$

$$= \boxed{0}$$

(b)[3] Evaluate $\lim_{x\to\infty} x^{1/(1+\ln x)}$. $\sim$ ∞^0 } indeterminate

$$\lim_{x\to\infty} x^{\frac{1}{1+\ln x}} = \lim_{x\to\infty} e^{\frac{\ln x}{1+\ln x}}$$

Now $\lim_{x\to\infty} \frac{\ln x}{1+\ln x} \sim \frac{\infty}{\infty}$

$$\overset{H}{=} \lim_{x\to\infty} \frac{\cancel{\frac{1}{x}}}{\cancel{\frac{1}{x}}} = 1$$

$$\therefore \lim_{x\to\infty} x^{\frac{1}{1+\ln x}} = e^1 = \boxed{e}$$

(c)[3] Evaluate $\lim_{x\to 1} \frac{\ln x}{\sin(\pi x)}$. $\sim \frac{0}{0}$

$$\overset{H}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{\pi\cos(\pi x)} = \boxed{\frac{-1}{\pi}}$$

Question 4:

(a)[3] Evaluate $\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$. $\sim$ "$\infty - \infty$"

$$= \lim_{x\to 1^+} \frac{x\ln x - x + 1}{(x-1)\ln x} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 1^+} \frac{\ln x + \cancel{x \frac{1}{x}} \cancel{-1}}{\ln x + (x-1)\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to 1^+} \frac{\ln x}{\ln x + 1 - \frac{1}{x}} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}} = \boxed{\frac{1}{2}}$$

(b)[4] Determine $f(x)$ if $f''(x) = \frac{-1}{x^2}$ where $f'(1) = 2$ and $f(1) = 4$.

$$f'(x) = \frac{1}{x} + C_1$$

$$f'(1) = 2 \Rightarrow 2 = \frac{1}{1} + C_1 \Rightarrow C_1 = 1$$

$$\therefore f'(x) = \frac{1}{x} + 1$$

$$\therefore f(x) = \ln|x| + x + C_2$$

$$f(1) = 4 \Rightarrow 4 = \underbrace{\cancel{\ln|1|}}_{0} + 1 + C_2 \Rightarrow C_2 = 3$$

$$\boxed{\therefore f(x) = \ln|x| + x + 3}$$

(c)[3] Determine the most general antiderivative of $f(x) = \frac{2}{\sqrt{1-x^2}} + \pi \sin x$.

$$\boxed{F(x) = 2\arcsin(x) - \pi\cos(x) + C}$$

Question 5:

(a)[4] The limit

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{i}{n}\right)^2-2\right]\left(\frac{1}{n}\right)$$

represents the area between the graph of $y=f(x)$ and the x-axis over a particular interval $[a,b]$. Identify the function $f(x)$ and the interval $[a,b]$.

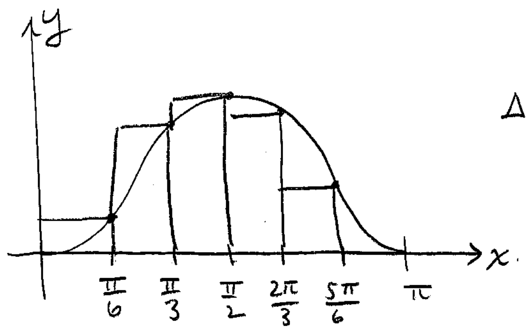
$\Delta x = \frac{1}{n}$, $x_i = 0 + i\left(\frac{1}{n}\right)$, so $a=0$, $b=1$

and $f(x_i) = x_i^2 - 2$, so $f(x) = x^2-2$

$\therefore$ $f(x) = x^2-2$

and $[a,b]=[0,1]$.

(b)[6] Use six subintervals and right endpoints to approximate the area under the graph of $y=\sin^2 x$ over the interval $[0,\pi]$.



$\Delta x = \frac{\pi}{6}$

$\therefore$ area $A \approx R_6 = \frac{\pi}{6}\left[\sin^2\left(\frac{\pi}{6}\right)+\sin^2\left(\frac{\pi}{3}\right)+\sin^2\left(\frac{\pi}{2}\right)+\sin^2\left(\frac{2\pi}{3}\right) + \sin^2\left(\frac{5\pi}{6}\right)+\sin^2(\pi)\right]$

$= \frac{\pi}{6}\left[\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2+1^2+\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2+0^2\right]$

$= \frac{\pi}{6}\cdot\frac{1+3+4+3+1}{4}$

$= \boxed{\frac{\pi}{2}}$