(1) [7] Determine

$$I = \int \tan^5 x \sec x \, dx$$

$$= \int \tan^4 x \, \sec x \tan x \, dx$$

$$= \int (\sec^2 x - 1)^2 \sec x \tan x \, dx$$

let $u = \sec x$

$du = \sec x \tan x \, dx$

$$\therefore \; I = \int (u^2 - 1)^2 \, du$$

$$= \int (u^4 - 2u^2 - 1) \, du$$

$$= \frac{u^5}{5} - \frac{2u^3}{3} - u + C$$

$$= \boxed{\frac{\sec^5 x}{5} - \frac{2\sec^3 x}{3} - \sec x + C}$$

(2) [8] Evaluate

$$\int_1^2 (\ln x)^2\, dx$$

For $I = \int (\ln x)^2 dx$, let $u = (\ln x)^2$, $dv = dx$

$du = \frac{2 \ln x}{x} dx$, $v = x$

$$\therefore \quad I = \int u\, dv$$

$$= uv - \int v\, du$$

$$= (\ln x)^2 x - \int x \frac{2 \ln x}{x}\, dx$$

$$= x(\ln x)^2 - 2 \underbrace{\int \ln x\, dx}$$

let $u = \ln x$, $dv = dx$

$du = \frac{1}{x} dx$, $v = x$

$$= x(\ln x)^2 - 2\left[x \ln x - \int \cancel{\tfrac{1}{x}} \cdot \cancel{x}\, dx \right]$$

$$= x(\ln x)^2 - 2x \ln x + 2x + C$$

$$\therefore \quad \int_1^2 (\ln x)^2 dx = \Big[I \Big]_1^2$$

$$= \left[2(\ln 2)^2 - (2)(2) \ln 2 + (2)(2) \right] - \left[1 \cdot \cancelto{0}{(\ln 1)^2} - \cancelto{0}{2(1)\ln(1)} + 2(1) \right]$$

$$= 2(\ln 2)^2 - 4 \ln 2 + 4 - 2$$

$$= 2\left((\ln 2)^2 - 2 \ln 2 + 1 \right)$$

$$= 2\left(\ln(2) - 1 \right)^2$$