

(1) [5] Determine $F''(2)$ if

$$F(x) = \int_1^x f(t) dt \quad \text{and} \quad f(t) = \int_1^{t^2} \frac{\sqrt{u^3-1}}{u} du$$

$$F'(x) = f(x) = \int_1^{x^2} \frac{\sqrt{u^3-1}}{u} du$$

$$F''(x) = f'(x) = \frac{\sqrt{(x^2)^3-1}}{x^2} \cdot (2x)$$

$$\therefore F''(2) = \frac{\sqrt{(2^2)^3-1}}{2^2} \cdot (2 \cdot 2)$$

$$= \boxed{\sqrt{63}}$$

(2) [5] Determine $\int \frac{a+bx^4}{5ax+bx^5} dx$.

$$u = 5ax + bx^5$$

$$du = (5a + 5bx^4) dx$$

$$\begin{aligned}\therefore \int \frac{a+bx^4}{5ax+bx^5} dx &= \frac{1}{5} \int \frac{1}{u} du \\ &= \frac{1}{5} \ln|u| + C\end{aligned}$$

$$= \boxed{\frac{1}{5} \ln|5ax+bx^5| + C}$$

(3) [5] Evaluate $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$x=1 \Rightarrow u=1$$

$$x=9 \Rightarrow u=3$$

$$\therefore \int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int_1^3 e^u du$$

$$= 2 [e^u]_1^3$$

$$= \boxed{2(e^3 - e)}$$