

(1) [5] Evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

$$\begin{aligned} \text{Let } u &= \sqrt{x} & x=1 &\Rightarrow u=1 \\ du &= \frac{1}{2\sqrt{x}} dx & x=4 &\Rightarrow u=2 \end{aligned}$$

$$\begin{aligned} \therefore \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx &= 2 \int_1^2 e^u du \\ &= 2 [e^u]_1^2 \\ &= \boxed{2(e^2 - e)} \end{aligned}$$

(2) [5] Determine $\int \frac{a + bx^3}{4ax + bx^4} dx$.

$$\begin{aligned} \text{Let } u &= 4ax + bx^4 \\ du &= (4a + 4bx^3) dx \end{aligned}$$

$$\begin{aligned} \therefore \int \frac{a + bx^3}{4ax + bx^4} dx &= \frac{1}{4} \int \frac{1}{u} du \\ &= \frac{1}{4} \ln |u| + C \\ &= \frac{1}{4} \ln |4ax + bx^4| + C \end{aligned}$$

(3). [5] Determine $F''(2)$ if

$$F(x) = \int_1^x f(t) dt \quad \text{and} \quad f(t) = \int_1^{t^2} \frac{\sqrt{1+u^3}}{u} du$$

$$F'(x) = f(x) = \int_1^{x^2} \frac{\sqrt{1+u^3}}{u} du$$

$$F''(x) = f'(x)$$

$$= \frac{\sqrt{1+(x^2)^3}}{x^2} \cdot (2x)$$

$$\begin{aligned} \therefore F''(2) &= \frac{\sqrt{1+(2^2)^3}}{\cancel{2^2}} \cdot \cancel{(2 \cdot 2)} \\ &= \boxed{\sqrt{65}} \end{aligned}$$