(1) [5] Evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

Let $u = \sqrt{x}$ $\quad x=1 \Rightarrow u=1$

$du = \frac{1}{2\sqrt{x}}\,dx$ $\quad x=4 \Rightarrow u=2$

$$\therefore \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^2 e^u\,du$$

$$= 2\left[e^u\right]_1^2$$

$$= \boxed{2(e^2-e)}$$

(2) [5] Determine $\int \frac{a+bx^3}{4ax+bx^4}\,dx$.

Let $u = 4ax+bx^4$

$du = (4a+4bx^3)\,dx$

$$\therefore \int \frac{a+bx^3}{4ax+bx^4}\,dx = \frac{1}{4}\int \frac{1}{u}\,du$$

$$= \frac{1}{4}\ln|u| + C$$

$$= \frac{1}{4}\ln|4ax+bx^4| + C$$

(3) [5] Determine $F''(2)$ if

$$F(x) = \int_1^x f(t)\,dt \quad \text{and} \quad f(t) = \int_1^{t^2} \frac{\sqrt{1+u^3}}{u}\,du$$

$$F'(x) = f(x) = \int_1^{x^2} \frac{\sqrt{1+u^3}}{u}\,du$$

$$F''(x) = f'(x)$$

$$= \frac{\sqrt{1+(x^2)^3}}{x^2} \cdot (2x)$$

$$\therefore F''(2) = \frac{\sqrt{1+(2^2)^3}}{2^2} \cdot (2 \cdot 2)$$

$$= \boxed{\sqrt{65}}$$