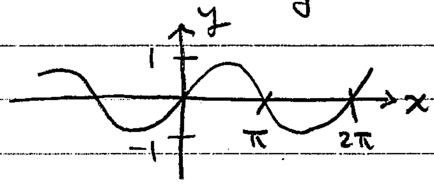
SUMMARY OF INVERSE TRIG FUNCTIONS

$f(x) = \sin^{-1}(x)$ or $\arcsin(x)$.

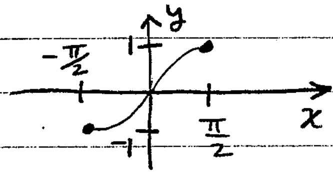
Definition

① start with $y = \sin x$



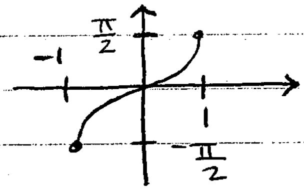
$y = \sin(x)$

② restrict domain



$y = \sin(x),\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

③ reflect about $y = x$



$y = \sin^{-1}(x)$.

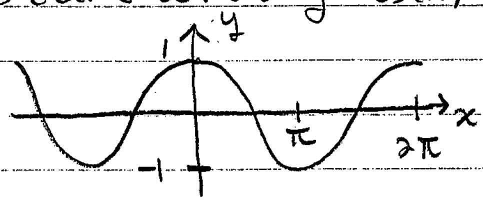
$\therefore\ f(x) = \sin^{-1}(x)$
$= \text{angle } y \text{ in } [-\frac{\pi}{2}, \frac{\pi}{2}] \text{ such that } \sin(y) = x$

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$$

$f(x) = \cos^{-1}(x)$ or $\arccos(x)$.

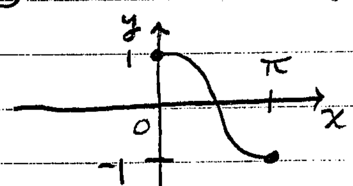
Definition:

① start with $y = \cos(x)$



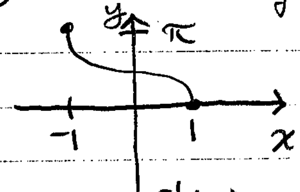
$y = \cos(x)$

② restrict domain



$y = \cos(x),\ 0 \le x \le \pi$

③ reflect about $y = x$



$y = \cos^{-1}(x)$

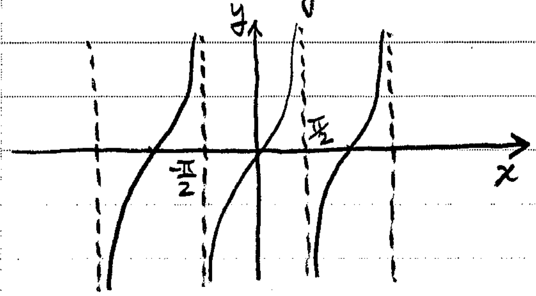
$\therefore\ f(x) = \cos^{-1}(x) = \text{angle } y \text{ in } [0, \pi] \text{ such that } \cos(y) = x$

$$\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}, \quad -1 < x < 1.$$

$f(x) = \tan^{-1}(x)$ or $\arctan(x)$

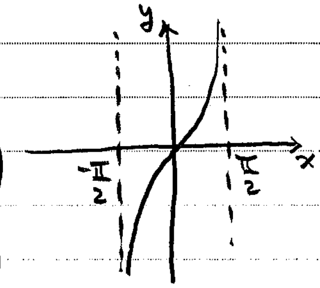
Definition:

① start with $y = \tan(x)$

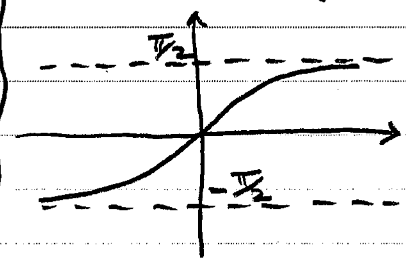


$y = \tan(x)$

② restrict domain



③ reflect about $y = x$



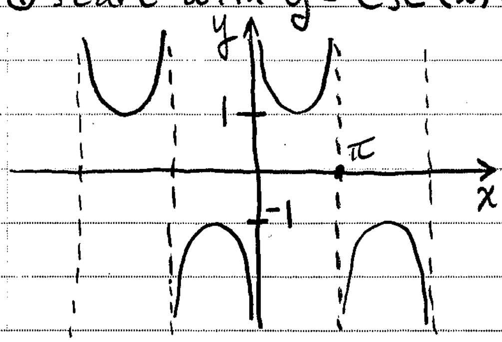
$y = \tan^{-1}(x)$

$\therefore f(x) = \tan^{-1}(x) =$ angle y in $(-\frac{\pi}{2}, \frac{\pi}{2})$ such that $\tan(y) = x$

$$\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}$$

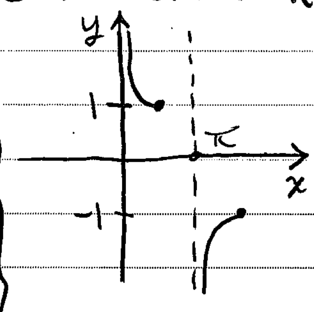
$f(x) = \csc^{-1}(x)$ or $\operatorname{arccsc}(x)$

① start with $y = \csc(x)$:

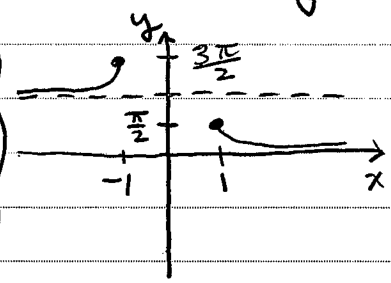


$y = \csc(x)$

② restrict domain



③ reflect about $y = x$



$y = \csc^{-1}(x)$

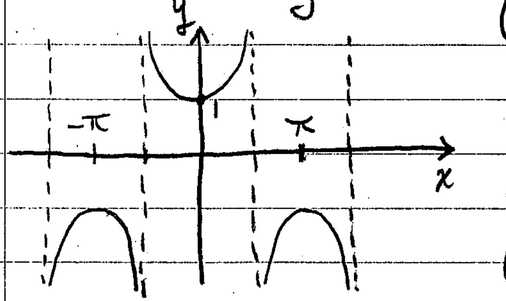
$\therefore f(x) = \csc^{-1}(x) =$ angle y in $(0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$
such that $\csc(y) = x$.

$$\frac{d}{dx}\left[\csc^{-1}(x)\right] = \frac{-1}{x\sqrt{x^2-1}}, \quad |x| > 1$$

$f(x) = \sec^{-1}(x)$ or $\operatorname{arcsec}(x)$

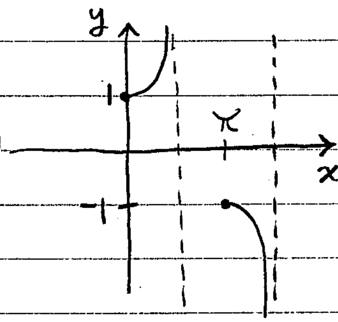
Definition :

① start with $y=\sec(x)$

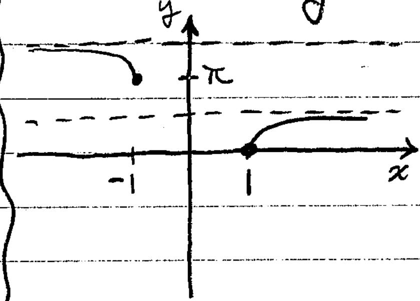


$y = \sec(x)$

② restrict domain



③ reflect about $y=x$



$y = \sec^{-1}(x)$

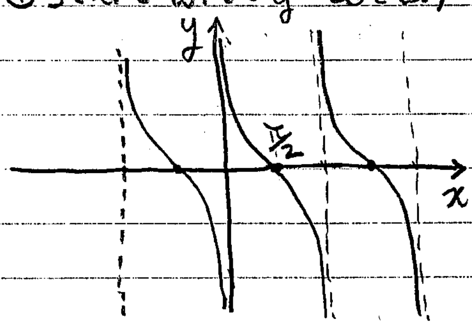
$\therefore\ f(x) = \sec^{-1}(x) =$ angle y in $[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$ such that $\sec(y) = x$

$$\frac{d}{dx}\left[\sec^{-1}(x)\right] = \frac{1}{x\sqrt{x^2-1}}, \quad |x| > 1$$

$f(x) = \cot^{-1}(x)$ or $\operatorname{arccot}(x)$

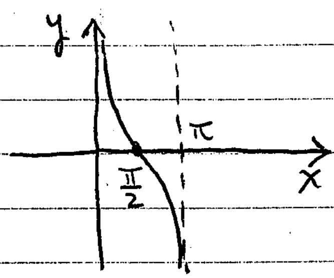
Definition :

① start with $y = \cot(x)$

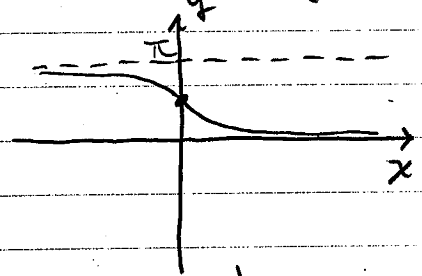


$y = \cot(x)$

② restrict domain



③ reflect about $y=x$



$y = \cot^{-1}(x)$

$\therefore\ f(x) = \cot^{-1}(x) =$ angle y in $(0, \pi)$ such that $\cot(y) = x$

$$\frac{d}{dx}\left[\cot^{-1}(x)\right] = \frac{-1}{1+x^2}$$