Question 1: A particle moves along a line with velocity function $v(t) = 3t^2 - 2t + k$ metres per second where k is a constant.

(a)[4] Determine k if the average velocity of the particle over the time period $[0, 5]$ is 15 metres per second.

$$\frac{1}{5-0}\int_0^5 3t^2 - 2t + k \, dt = 15$$

$$\frac{1}{5}\left[\frac{\cancel{3}t^3}{\cancel{3}} - \frac{\cancel{2}t^2}{\cancel{2}} + kt\right]_0^5 = 15$$

$$\frac{125 - 25 + 5k}{5} = 15$$

$$20 + k = 15$$

$$\boxed{k = -5}$$

(b)[2] Using the k value determined in (a), determine the total displacement of the particle over the time period $[0, 5]$.

$$\text{total displacement} = \int_0^5 3t^2 - 2t - 5 \, dt$$

$$= \left[t^3 - t^2 - 5t\right]_0^5$$

$$= 125 - 25 - 25$$

$$= \boxed{75 \text{ m}}$$

(c)[4] Again using the k value determined in (a), is there a time $t_1 > 0$ at which the total displacement will be zero?

$$\int_0^{t_1} 3t^2 - 2t - 5 \, dt = 0$$

$$t_1^3 - t_1^2 - 5t_1 = 0$$

$$t_1(t_1^2 - t_1 - 5) = 0$$

$$\cancel{t_1 = 0} \qquad t_1 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-5)}}{2}$$

$$\boxed{t_1 = \frac{1 + \sqrt{21}}{2}}, \quad \cancel{t_1 = \frac{1 - \sqrt{21}}{2}}$$

Question 2:

(a)[3] Determine $g'(1)$ where

$$g(x) = \int_{2x}^{3x+1} \sin(t^2\pi/4)\,dt$$

$$g'(x) = \sin\left[(3x+1)^2\frac{\pi}{4}\right](3) - \sin\left[(2x)^2\frac{\pi}{4}\right](2)$$

$$\therefore\ g'(1) = \cancel{\sin\left[16\frac{\pi}{4}\right]}^{\to 0}(3) - \cancel{\sin\left[4\frac{\pi}{4}\right]}^{\to 0}(2)$$

$$= \boxed{0}$$

(b)[3] Determine $\int \tan x \ln(\cos x)\,dx = \int \frac{\sin x}{\cos x} \cdot \ln(\cos x)\,dx$

Let $u = \ln(\cos x)$

$$du = \frac{1}{\cos x} \cdot (-\sin x)\,dx$$

$$\therefore \int \tan x \ln(\cos x)\,dx = -\int u\,du$$

$$= -\frac{u^2}{2} + C$$

$$= \boxed{-\frac{[\ln(\cos x)]^2}{2} + C}$$

(c)[4] Evaluate $\int_0^1 x\sqrt{1-x}\,dx$

Let $u = 1-x$, $du = -dx$

$x = 0 \Rightarrow u = 1$, $x = 1 \Rightarrow u = 0$

$$\therefore \int_0^1 x\sqrt{1-x}\,dx = -\int_1^0 (1-u)\,u^{1/2}\,du$$

$$= \int_0^1 u^{1/2} - u^{3/2}\,du$$

$$= \frac{2}{3}\left[u^{3/2}\right]_0^1 - \frac{2}{5}\left[u^{5/2}\right]_0^1$$

$$= \frac{2}{3} - \frac{2}{5} = \boxed{\frac{4}{15}}$$

Question 3 [10]: Evaluate $\int_0^1 (x^2+1)e^{-x}\,dx$

For $I = \int (x^2+1)e^{-x}\,dx$, let $u = x^2+1$, $dv = e^{-x}dx$

$du = 2x\,dx$, $v = -e^{-x}$

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= (x^2+1)(-e^{-x}) + \underbrace{\int 2x e^{-x}\,dx}$$

$u = 2x$, $dv = e^{-x}dx$

$du = 2dx$, $v = -e^{-x}$

$$= -(x^2+1)e^{-x} - 2xe^{-x} + \int 2e^{-x}\,dx$$

$$= -(x^2+1)e^{-x} - 2xe^{-x} - 2e^{-x} + C.$$

$$\therefore \int_0^1 (x^2+1)e^{-x}\,dx = \left[-(x^2+1)e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^1$$

$$= \left(-2e^{-1} - 2e^{-1} - 2e^{-1}\right) - (-1-0-2)$$

$$= \boxed{3-6e^{-1}}$$

Question 4 [10]: Determine $\int x^3\sqrt{9-x^2}\,dx$

Let $x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$\therefore \int x^3\sqrt{9-x^2}\,dx$$

$$= \int 3^3\sin^3\theta\sqrt{9-9\sin^2\theta}\;3\cos\theta\,d\theta$$

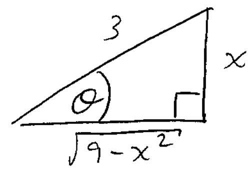
$$= 3^5\int \sin^3\theta\cos^2\theta\,d\theta$$

$$= 3^5\int(1-\cos^2\theta)\cos^2\theta\sin\theta\,d\theta \quad \left\{ \begin{array}{l} \text{let } u=\cos\theta \\ du=-\sin\theta\,d\theta \end{array} \right.$$

$$= -3^5\int(1-u^2)u^2\,du$$

$$= -3^5\int u^2-u^4\,du$$

$$= -3^5\left[\frac{u^3}{3}-\frac{u^5}{5}\right]+C$$



$$= -3^5\left[\frac{\cos^3\theta}{3}-\frac{\cos^5\theta}{5}\right]+C$$

$$= -3^5\left[\frac{(\sqrt{9-x^2})^3}{3^4}-\frac{(\sqrt{9-x^2})^5}{5\cdot 3^5}\right]+C$$

$$= \boxed{\frac{(9-x^2)^{5/2}}{5} - 3(9-x^2)^{3/2}+C}$$

Question 5 [10]: Determine $\int \frac{25x}{(x-3)(x+2)^2}\,dx$

$$\frac{25x}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$= \frac{A(x+2)^2 + B(x-3)(x+2) + C(x-3)}{(x-3)(x+2)^2}$$

$$= \frac{Ax^2 + 4Ax + 4A + Bx^2 - Bx - 6B + Cx - 3C}{(x-3)(x+2)^2}$$

$$= \frac{(A+B)x^2 + (4A-B+C)x + (4A-6B-3C)}{(x-3)(x+2)^2}$$

$\therefore\ A+B=0 \implies A=-B$

$4A-B+C=25 \implies C = 25+B-4A = 25+5B$

$4A-6B-3C=0 \implies 4(-B)-6B-3(25+5B)=0$

$-4B-6B-75-15B=0$

$-25B=75$

$B=-3$

$\therefore\ A=3$

$C = 25+5(-3) = 10$

$$\therefore \int \frac{25x}{(x-3)(x+2)^2}\,dx = \int \frac{3}{x-3}\,dx + \int \frac{-3}{x+2}\,dx + \int \frac{10}{(x+2)^2}\,dx$$

$$= \boxed{3\ln|x-3| - 3\ln|x+2| - \frac{10}{(x+2)} + C}$$