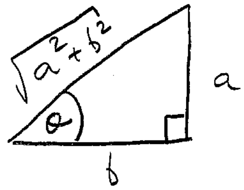
Question 1:

(a)[3] Simplify csc(arctan(a/b)). Your final answer should not contain any trigonometric or inverse trigonometric functions.

Let $\theta = \arctan\left(\frac{a}{b}\right)$, so $\tan\theta = \frac{a}{b}$;



$\therefore \csc\left(\arctan\left(\frac{a}{b}\right)\right)$

$= \csc(\theta)$

$= \boxed{\frac{\sqrt{a^2+b^2}}{a}}$

(b)[4] Determine an equation of the tangent line to the curve $y = 3\arccos(x/2)$ at the point where $x = 1$.

At $x = 1$, $y = 3\arccos\left(\frac{1}{2}\right) = 3\left(\frac{\pi}{3}\right) = \pi$, so $(1, \pi)$ is a point on the line.

The slope of the line is $\left.\frac{dy}{dx}\right|_{x=1} = \left.\frac{-3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}\right|_{x=1}$

$= \frac{-3}{\sqrt{1-\left(\frac{1}{2}\right)^2}} \cdot \frac{1}{2}$

$= -\sqrt{3}$

$\therefore$ Equation is $\boxed{y - \pi = -\sqrt{3}\,(x-1)}$

(c)[3] Determine $\lim_{x\to 0^+} \arctan(1/\sqrt{x})$.

As $x \to 0^+$, $\frac{1}{\sqrt{x}} \to \infty$, so $\arctan\left(\frac{1}{\sqrt{x}}\right) \to \frac{\pi}{2}$

$\therefore \lim_{x\to 0^+} \arctan\left(\frac{1}{\sqrt{x}}\right) = \boxed{\frac{\pi}{2}}$

Question 2:

(a)[4] Let $f(x) = \sinh(x + \sinh^2 x)$. Determine $f'(0)$.

$$f'(x) = \cosh\left(x + \sinh^2 x\right)\left(1 + 2\sinh(x)\cosh(x)\right)$$

$$f'(0) = \cosh\left(0 + \sinh^2(0)\right)\left(1 + 2\sinh(0)\cosh(0)\right)$$

$$0 \qquad 0$$

$$= \boxed{1}$$

(b)[6] Determine the two values of x at which the tangent lines to the curve $y = \sinh x$ have slope 2 .

Solve $\frac{d}{dx}\left[\sinh(x)\right] = 2$

$$\Rightarrow \quad \cosh(x) = 2.$$

$$\frac{e^x + e^{-x}}{2} = 2$$

$$e^x + e^{-x} = 4$$

$$e^{2x} + 1 = 4e^x$$

$$e^{2x} - 4e^x + 1 = 0$$

$$(e^x)^2 - 4(e^x) + 1 = 0$$

$$\therefore \; e^x = \frac{+4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

$$\therefore \; \boxed{x = \ln(2+\sqrt{3}), \quad x = \ln(2-\sqrt{3})}$$

Question 3:

(a)[3] Evaluate $\lim_{x\to 1} \frac{\ln x}{\sin(\pi x)}$. $\sim$ "$\frac{0}{0}$"

$$\overset{H}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{\pi\cos(\pi x)}$$

$$= \boxed{\frac{-1}{\pi}}$$

(b)[4] Evaluate $\lim_{x\to -\infty} x^2e^{2x}$. $\sim$ "$\infty \cdot 0$" : indeterminate

$$\lim_{x\to -\infty} x^2 e^{2x} = \lim_{x\to -\infty} \frac{x^2}{e^{-2x}} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to -\infty} \frac{2x}{-2e^{-2x}} \sim \text{"}\frac{-\infty}{-\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to -\infty} \frac{2}{4e^{-2x}} = \boxed{0}$$

(c)[3] Evaluate $\lim_{x\to\infty} x^{1/(1+\ln x)}$. $\sim$ "∞^0" : indeterminate

$$\lim_{x\to\infty} x^{\frac{1}{1+\ln x}} = \lim_{x\to\infty} e^{\left(\frac{\ln x}{1+\ln x}\right)}$$

Consider $\lim_{x\to\infty} \frac{\ln x}{1+\ln x} \sim \frac{\infty}{\infty}$

$$\overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x}} = 1$$

$$\therefore \lim_{x\to\infty} x^{\frac{1}{1+\ln x}} = e^1 = \boxed{e}$$

Question 4:

(a)[3] Evaluate $\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$. $\sim$ "$\infty-\infty$"

$$= \lim_{x\to 1^+} \frac{x\ln x - x + 1}{(x-1)\ln x} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 1^+} \frac{\ln x + \cancel{x\frac{1}{x}-1}}{\ln x + (x-1)\frac{1}{x}}$$

$$= \lim_{x\to 1^+} \frac{\ln x}{\ln x + 1 - \frac{1}{x}} \sim \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}} = \boxed{\frac{1}{2}}$$

(b)[3] Determine the most general antiderivative of $f(x) = \frac{2}{\sqrt{1-x^2}} + \pi \sin x$.

$$\boxed{F(x) = 2\arcsin(x) - \pi\cos(x) + C}$$

(c)[4] Determine $f(x)$ if $f''(x) = \frac{-1}{x^2}$ where $f'(1) = 2$ and $f(1) = 4$.

$$f'(x) = \frac{1}{x} + C_1$$

$$f'(1) = 2 \Rightarrow 2 = \frac{1}{1} + C_1 \Rightarrow C_1 = 1$$

$$\therefore f'(x) = \frac{1}{x} + 1$$

$$\therefore f(x) = \ln|x| + x + C_2$$

$$f(1) = 4 \Rightarrow 4 = \underbrace{\cancel{\ln|1|}}_{0} + 1 + C_2 \Rightarrow C_2 = 3$$

$$\boxed{\therefore f(x) = \ln|x| + x + 3}$$

Question 5:

(a)[4] The limit

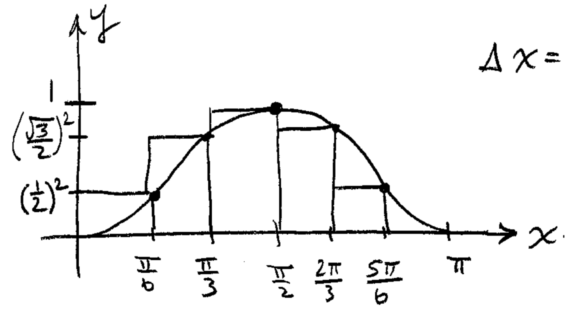
$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[1+\left(\frac{i}{n}\right)^2\right]\left(\frac{1}{n}\right)$$

represents the area between the graph of $y = f(x)$ and the x-axis over a particular interval $[a, b]$. Identify the function $f(x)$ and the interval $[a, b]$.

Here $\Delta x = \frac{1}{n}$, $x_i = 0 + i\left(\frac{1}{n}\right)$, so $a = 0$, $b = 1$,

$f(x_i) = 1 + x_i^2$, ~~so~~ $\boxed{f(x) = 1 + x^2 \quad [a,b] = [0,1]}$

(b)[6] Use six subintervals and right endpoints to approximate the area under the graph of $y = \sin^2 x$ over the interval $[0, \pi]$.



$\Delta x = \frac{\pi}{6}$.

Area $A \approx R_6 = \frac{\pi}{6}\Big[\sin^2\left(\frac{\pi}{6}\right) + \sin^2\left(\frac{\pi}{3}\right) + \sin^2\left(\frac{\pi}{2}\right) + \sin^2\left(\frac{2\pi}{3}\right) + \sin^2\left(\frac{5\pi}{6}\right) + \sin^2(\pi)\Big]$

$= \frac{\pi}{6}\left[\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 0^2\right]$

$= \frac{\pi}{6}\left[\frac{1+3+4+3+1}{4}\right]$

$= \boxed{\frac{\pi}{2}}$