

(1) [3] Determine $\lim_{x \rightarrow -\infty} \tanh x$.

$$\begin{aligned} \lim_{x \rightarrow -\infty} \tanh(x) &= \lim_{x \rightarrow -\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\cancel{e^{-x}}(e^{2x} - 1)}{\cancel{e^{-x}}(e^{2x} + 1)} \\ &= \boxed{-1} \end{aligned}$$

(2) [4] Differentiate $h(t) = \coth \sqrt{1-t^2}$.

$$\begin{aligned} h'(t) &= -\operatorname{csch}^2 \left([1-t^2]^{\frac{1}{2}} \right) \cdot \frac{1}{2} (1-t^2)^{-\frac{1}{2}} (-2t) \\ &= \boxed{\frac{t \operatorname{csch}^2(\sqrt{1-t^2})}{\sqrt{1-t^2}}} \end{aligned}$$

(3) [4] Simplify $\cosh(\ln 2)$.

$$\begin{aligned}
 \cosh(\ln 2) &= \frac{e^{\ln 2} + e^{-\ln 2}}{2} \\
 &= \frac{e^{\ln 2} + e^{\ln(2^{-1})}}{2} \\
 &= \frac{2 + \frac{1}{2}}{2} \\
 &= \boxed{\frac{5}{4}}
 \end{aligned}$$

(4) [4] Evaluate the following limit. Be sure to indicate when you are using L'Hospital's Rule and also state the type of indeterminate form you found which justifies the use of the rule.

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} \sim \frac{0}{0}$$

$$\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} \sim \frac{0}{0}$$

$$\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x}{2}$$

$$= \boxed{\frac{1}{2}}$$