Question 1:

(a)[5] Determine $\displaystyle\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}\,\sin\left(\sqrt{x}\right)}\,dx$

(b)[5] Determine $\displaystyle\int \sec^3 x\,\tan^3 x\,dx$

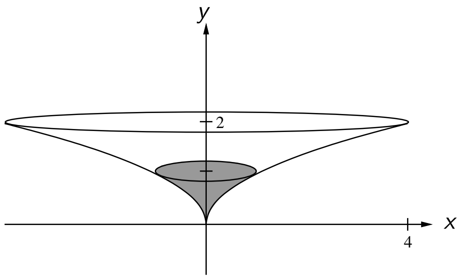
Question 2:

(a)[5] Compute) $\displaystyle\int_0^{1/2} \arctan(2x)\,dx$

(b)[5] Determine $\displaystyle\int \frac{1}{x(x-1)^2}\,dx$

Question 3:

(a)[5 points] A vessel is formed by rotating the curve $y = \sqrt{x}$ about the y-axis as shown below. The vessel has a top radius of 4 m and a depth of 2 m. If the vessel is initially filled with water to a depth of 1 m, how much work is required to empty the vessel by pumping the water up and over the top rim? Recall that the density of water is $\rho = 1000$ kg/m^3 and acceleration due to gravity is $g = 9.8$ m/s^2.



(b)[5 points] Determine the total volume of the vessel described in part (a).

Question 4:

(a)[5] Determine the length of the curve $y = \ln(\cos x)$, $0 \leq x \leq \pi/3$

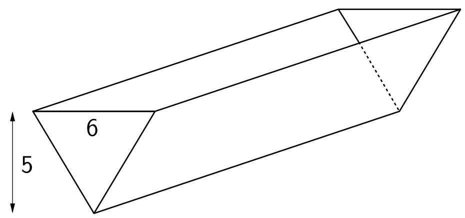
(b)[5] Solve the following differential equation:

$$(1 + \cos x)y' = e^{-y} \sin x, \qquad y(0) = 0$$

You may leave your solution in implicit form (it is not necessary to isolate y in your final answer.)

Question 5:

(a)[5 points] A large 5 m deep trough has triangular ends of top width 6 m. If the trough is full of water determine the hydrostatic force (force due to water pressure) exerted on one end of the trough. Recall that the density of water is $\rho = 1000$ kg/m^3, acceleration due to gravity is $g = 9.8$ m/s^2, and pressure P as a function of depth h is $P(h) = \rho g h$.



(b)[5 points] Determine the first three nonzero terms of the Maclaurin series for $f(x) = x \sin(x^4)$.

Question 6:

(a)[5 points] Use a Maclaurin series (not L'Hospital's Rule) to evaluate the limit

$$\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$$

(b)[2 points] Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$ converges or diverges.

(c)[3 points] Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(n+1)}$ converges or diverges.