

(1) [8] Evaluate

$$\int_{\sqrt{2}}^2 \frac{1}{t^2 \sqrt{t^2 - 1}} dt$$

$$\text{Let } t = \sec \theta$$

$$dt = \sec \theta \tan \theta d\theta$$

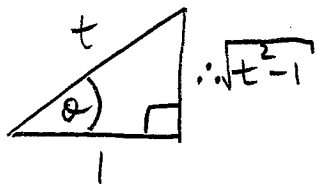
$$\therefore I = \int \frac{1}{t^2 \sqrt{t^2 - 1}} dt$$

$$= \int \frac{1}{\sec^2 \theta \sqrt{\sec^2 \theta - 1}} \sec \theta \tan \theta d\theta$$

$$= \int \frac{1}{\sec^2 \theta \cancel{\tan \theta}} \cancel{\sec \theta \tan \theta} d\theta$$

$$= \int \cos \theta d\theta$$

$$= \sin \theta + C$$



$$\therefore I = \sin \theta + C$$

$$= \frac{\sqrt{t^2 - 1}}{t} + C.$$

$$\therefore \int_{\sqrt{2}}^2 \frac{1}{t^2 \sqrt{t^2 - 1}} dt$$

$$= \left[ \frac{\sqrt{t^2 - 1}}{t} \right]_{\sqrt{2}}^2$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{6} - 2}{2\sqrt{2}}}$$

(2) [7] Determine

$$I = \int \frac{x-9}{(x+5)(x-2)} dx$$

$$\begin{aligned} \frac{x-9}{(x+5)(x-2)} &= \frac{A}{x+5} + \frac{B}{x-2} \\ &= \frac{Ax - 2A + Bx + 5B}{(x+5)(x-2)} \\ &= \frac{(A+B)x - 2A + 5B}{(x+5)(x-2)} \end{aligned}$$

$$\therefore A+B = 1$$

$$-2A + 5B = -9$$

$$\therefore -2A + 5(1-A) = -9$$

$$-7A = -14$$

$$A = 2$$

$$B = -1$$

$$\therefore I = \int \frac{2}{x+5} - \frac{1}{x-2} dx$$

$$= \boxed{2 \ln|x+5| - \ln|x-2| + C.}$$