

(1) [8] Evaluate $\int_0^{\pi} t \sin(3t) dt$.

$$\begin{aligned} \text{Let } w &= 3t, & t=0 &\Rightarrow w=0 \\ dw &= 3 dt & t=\pi &\Rightarrow w=3\pi \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\pi} t \sin(3t) dt &= \frac{1}{3} \int_0^{3\pi} \left(\frac{w}{3}\right) \sin(w) dw \\ &= \frac{1}{9} \int_0^{3\pi} w \sin(w) dw. \end{aligned}$$

$$\begin{aligned} \text{For } I = \int w \sin(w) dw : & \quad u = w \quad ; \quad dv = \sin(w) dw \\ du = dw & \quad v = -\cos(w) \end{aligned}$$

$$\begin{aligned} \therefore I &= \int u dv = uv - \int v du \\ &= (w)(-\cos w) - \int -\cos(w) dw \\ &= -w \cos w + \sin(w) + C \end{aligned}$$

$$\begin{aligned} \therefore \frac{1}{9} \int_0^{3\pi} w \sin(w) dw &= \frac{1}{9} \left[-w \cos(w) + \sin(w) \right]_0^{3\pi} \\ &= \frac{1}{9} \left[-3\pi \cos(3\pi) + \sin(3\pi) \right] - \left[0 + \sin(0) \right] \\ &= \boxed{\frac{\pi}{3}} \end{aligned}$$

(2) [7] Determine $\int \tan^5(x) \sec^4(x) dx$.

$$\int \tan^5 x \sec^4 x dx$$

$$= \int \tan^4 x \sec^3 x \sec x \tan x dx$$

$$= \int (\sec^2 x - 1)^2 \sec^3 x \sec x \tan x dx \quad \left. \begin{array}{l} \text{let } u = \sec x \\ du = \sec x \tan x dx \end{array} \right\}$$

$$= \int (u^2 - 1)^2 u^3 du$$

$$= \int (u^7 - 2u^5 + u^3) du$$

$$= \frac{u^8}{8} - \frac{2u^6}{6} + \frac{u^4}{4} + C$$

$$= \frac{\sec^8 x}{8} - \frac{1}{3} \sec^6 x + \frac{1}{4} \sec^4 x + C$$

$$\text{or } \frac{\tan^8 x}{8} + \frac{\tan^6 x}{6} + C$$