(1) [10] Use the definition of the definite integral in the form

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

to determine

$$\int_0^2 (2-x^2)\,dx$$

You may wish to recall the following special sums:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Here $[a,b] = [0,2]$, $f(x) = 2-x^2$,

$\Delta x = \frac{2-0}{n} = \frac{2}{n}$

$x_i = 0 + i\Delta x = i\left(\frac{2}{n}\right)$

$$\therefore \int_0^2 (2-x^2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2-\left(i\cdot\frac{2}{n}\right)^2\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{4}{n} - \left(\frac{8}{n^3}\right) i^2\right]$$

$$= \lim_{n\to\infty} \left[\sum_{i=1}^{n}\left(\frac{4}{n}\right) - \left(\frac{8}{n^3}\right)\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\left(\frac{4}{n}\right)n - \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \left[4 - \frac{4}{3}\cdot\cancel{\frac{n}{n}}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}\right]$$

$$= 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$$

(2) [5] Now check your answer from question (1) by using the Fundamental Theorem of Calculus to evaluate

$$\int_0^2 (2 - x^2)\, dx$$

$$= \left[2x - \frac{x^3}{3}\right]_0^2$$

$$= \left((2)(2) - \frac{(2)^3}{3}\right) - \left((2)(0) - \frac{(0)^3}{3}\right)$$

$$= 4 - \frac{8}{3}$$

$$= \boxed{\frac{4}{3}}.$$