

(1) [5] The following limit represents the area under the graph of a certain function  $f(x)$  over a certain interval  $[a, b]$ . Determine  $f(x)$  and  $[a, b]$ :

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{6n} \tan\left(\frac{i\pi}{6n}\right)$$

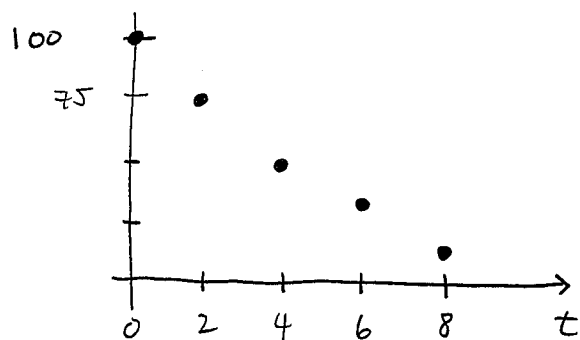
$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \tan\left(\underbrace{i \frac{(\pi/6)}{n}}_{x_i = i\Delta x}\right) \underbrace{\frac{(\pi/6)}{n}}_{\Delta x}$$

$$\therefore f(x) = \tan(x)$$

$$[a, b] = \left[0, \frac{\pi}{6}\right]$$

(2) [5] The population of a certain town is growing at a decreasing rate of  $r(t)$  people per year. Using the following data for  $r(t)$ , give lower and upper estimates for the change in population over the time period  $t = 0$  to  $t = 8$ . State units with your answer.

|                    |     |    |    |    |    |
|--------------------|-----|----|----|----|----|
| $t$ (yrs)          | 0   | 2  | 4  | 6  | 8  |
| $r(t)$ (people/yr) | 100 | 75 | 50 | 36 | 10 |



Let  $P$  = population change.

$$\text{Then } P < (100)(2) + (75)(2) + (50)(2) + (36)(2)$$

$$\therefore P < 522 \text{ people.}$$

$$P > (75)(2) + (50)(2) + (36)(2) + (10)(2)$$

$$P > 342 \text{ people.}$$

$$\therefore 342 < P < 522 \text{ people}$$