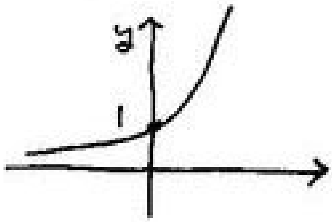
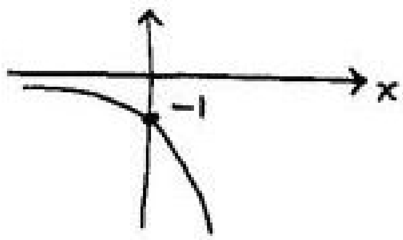
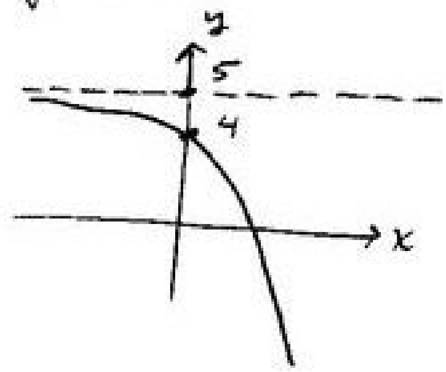
① $y = 2^x$



$y = -2^x$



$y = 5 - 2^x$



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② $e^{x^2-3} = e^{2x}$

$x^2 - 3 = 2x$

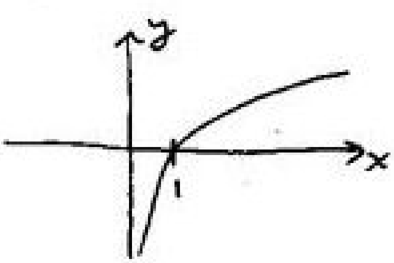
$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

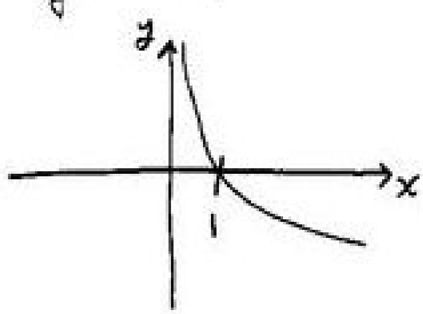
$\boxed{\therefore\ x = 3,\ x = -1}$

---

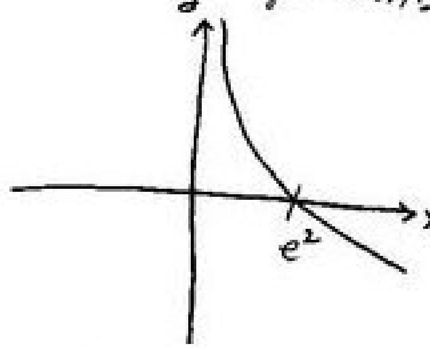
③ $y = \ln(x)$



$y = -\ln x$



$y = -\ln(x) + 2$



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④ $\log_5 75 - \log_5 3 = \log_5\left(\frac{75}{3}\right) = \log_5(25) = \boxed{2}$

---

⑤ $\ln x - 4\left[\ln(x+2) + \ln(x-2)\right]$

$= \ln x - 4 \ln\left(\frac{x+2}{x-2}\right)$

$= \ln x - \ln\left(\frac{x+2}{x-2}\right)^4$

$= \ln\left[\frac{x}{\left(\frac{x+2}{x-2}\right)^4}\right] = \ln\left(\frac{x(x-2)^4}{(x+2)^4}\right).$

⑥ $\frac{\pi}{a}$ radians $= \frac{\pi}{a} \cdot \frac{180°}{\pi \text{ radians}} = \boxed{\frac{180°}{a}}$

---

⑦ $\cos\left(\frac{95\pi}{6} - 10\pi\right) = \cos\left(\frac{95\pi}{6}\right) = \cos\left(\frac{96\pi - \pi}{6}\right) = \cos\left(16\pi - \frac{\pi}{6}\right)$

$= \cos\left(-\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}.$

$\tan\left(-\frac{50\pi}{3}\right) = \tan\left(\frac{-51\pi + \pi}{3}\right) = \tan\left(-17\pi + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right)$

$= \frac{\sin(\pi/3)}{\cos(\pi/3)}$

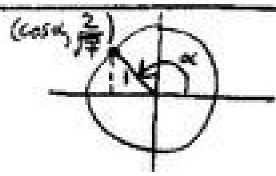
$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$\therefore \cos\left(\frac{95\pi}{6} - 10\pi\right) \tan\left(-\frac{50\pi}{3}\right)$

$= \frac{\sqrt{3}}{2} \cdot \sqrt{3} = \boxed{\frac{3}{2}}$

---

⑧ $\sin\alpha = \frac{2}{\sqrt{7}}$, $\frac{\pi}{2} \le \alpha \le \pi$:



$\therefore \cos\alpha = -\sqrt{1^2 - \left(\frac{2}{\sqrt{7}}\right)^2} = -\sqrt{\frac{3}{7}}$

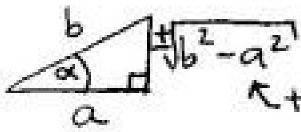
by pythagoras

(negative square root since $\cos\alpha$ must be negative)

$\therefore \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{2/\sqrt{7}}{-\sqrt{3/7}} = -\frac{2}{\sqrt{3}}.$

---

⑨ $\cos\alpha = \frac{a}{b}$



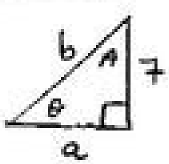
two possibilities depending on α

$\therefore \csc\alpha = \frac{b}{\pm\sqrt{b^2 - a^2}}$

$\therefore \boxed{\csc^2\alpha = \frac{b^2}{b^2 - a^2}}$

---

⑩



$\sec\theta = \sqrt{5}$

$\therefore \frac{b}{a} = \sqrt{5}$

$\therefore b = \sqrt{5}a$

Also, $a^2 + 7^2 = b^2$

$\therefore a^2 + 7^2 = (\sqrt{5}a)^2$

$a^2 + 49 = 5a^2$

$49 = 4a^2$

$\boxed{\begin{array}{l} \therefore a = \sqrt{\frac{49}{4}} = \frac{7}{2} \\ \therefore b = \sqrt{5}\,\frac{7}{2} \end{array}}$

$\therefore \theta = \tan^{-1}\left(\frac{7}{7/2}\right)$

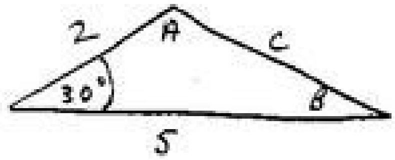
$= \tan^{-1}(2)$

$\therefore \boxed{\theta \doteq 63.4°}$

$\therefore A = 90 - \theta$

$\boxed{A \doteq 26.6°}$

(11)

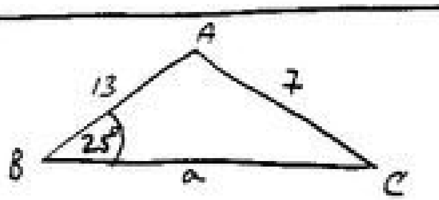


$$c^2 = 2^2 + 5^2 - 2(2)(5)\cos(30°)$$
$$= 4 + 25 - 20\frac{\sqrt{3}}{2}$$
$$= 29 - 10\sqrt{3}$$
$$\therefore c = \sqrt{29 - 10\sqrt{3}} \doteq \boxed{3.4}$$

$$\left.\frac{\sin(30°)}{c} = \frac{\sin B}{2}\right\} \therefore B = \sin^{-1}\left[\frac{2 \cdot \sin(30°)}{c}\right]$$
$$= \sin^{-1}\left[\frac{2 \cdot \frac{1}{2}}{\sqrt{29 - 10\sqrt{3}}}\right]$$
$$\boxed{B \doteq 17.0°}$$
$$\therefore A = 180 - B - 30 \doteq \boxed{133.0°}$$

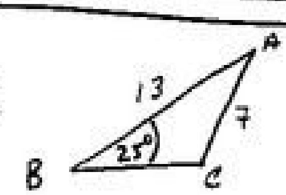
---

(12)



$$\frac{\sin(25)}{7} = \frac{\sin C}{13}$$
$$\therefore C = \sin^{-1}\left[\frac{13\sin(25)}{7}\right]$$
$$\boxed{C \doteq 51.7°}$$
$$\boxed{\therefore A = 180 - 25 - C \doteq 103.3°}$$
$$\therefore a^2 = 13^2 + 7^2 - 2(13)(7)\cos(A)$$
$$\therefore a = \sqrt{218 - 182\cos(A)}$$
$$\boxed{a \doteq 16.1}$$

or



$$\frac{\sin(25)}{7} = \frac{\sin(C)}{13}$$
$$\therefore C = 180 - \sin^{-1}\left[\frac{13\sin(25)}{7}\right] \Bigg\} \text{Notice! Since } 90 \leq C \leq 180$$
$$\boxed{\therefore C \doteq 128.3°}$$
$$\therefore A = 180 - 25 - C$$
$$\boxed{A \doteq 26.7°}$$
$$a^2 = 13^2 + 7^2 - 2(13)(7)\cos(A)$$
$$\therefore a = \sqrt{218 - 182\cos(A)}$$
$$\boxed{a \doteq 7.4}$$

---

(13)

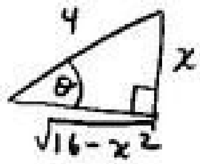
$$\sin\left(\frac{\pi}{10}\right) = \frac{\sqrt{5}-1}{4}$$

$$\therefore \left[4^2 - (\sqrt{5}-1)^2\right]^{1/2} = \sqrt{10 + 2\sqrt{5}}$$
$$\therefore \cos\left(\frac{\pi}{10}\right) = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

using $\sin(2\theta) = 2\sin\theta\cos\theta$,

$$\sin\left(\frac{\pi}{5}\right) = 2\sin\left(\frac{\pi}{10}\right)\cos\left(\frac{\pi}{10}\right) = 2\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right) = \boxed{\frac{(\sqrt{5}-1)\sqrt{10+2\sqrt{5}}}{8}}$$

(14) $\sin\theta = \frac{x}{4}$



$\left.\right\} \therefore \cos\theta = \frac{\sqrt{16-x^2}}{4}$

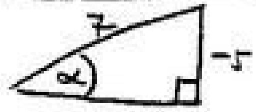
using $\sin(2\theta) = 2\sin\theta\cos\theta$,

$$\sin(2\theta) = 2\cdot\frac{x}{4}\cdot\frac{\sqrt{16-x^2}}{4}$$

$$= \boxed{\frac{x\sqrt{16-x^2}}{8}}$$

---

(15) Let $\alpha = \sin^{-1}\left(\frac{-5}{7}\right)$, so $\sin\alpha = \frac{-5}{7}$



$\therefore \sqrt{49-25} = \sqrt{24}$.

$$\therefore \tan\left[\sin^{-1}\left(\frac{-5}{7}\right)\right] = \tan(\alpha) = \boxed{\frac{-5}{\sqrt{24}}}$$

---

(16) $$\sin^{-1}\left[\sin\left(-\frac{81\pi}{10}\right)\right] = \sin^{-1}\left[\sin\left(\frac{-80\pi-\pi}{10}\right)\right]$$

$$= \sin^{-1}\left[\sin\left(-8\pi - \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right]$$

$$= \boxed{\frac{-\pi}{10}} \quad \text{since} \quad -\frac{\pi}{2} \le -\frac{\pi}{10} \le \frac{\pi}{2}$$

---

(17)

A is 5×1, B is 5×1, so $(A+B)$ is 5×1, and $(A+B)^T$ is 1×5.

$\therefore$ AI_1 is 5×1; $AI_1(A+B)^T$ is 5×5; $AI_1(A+B)^T I_5$ is 5×5

$$\boxed{\therefore\ AI_1(A+B)^T I_5 B \text{ is } 5\times1}$$

---

(18) $$\left(\tfrac{1}{2}A + \tfrac{1}{2}B\right)(2A-2B) - BA = \cancel{\tfrac{1}{2}}(A+B)\cancel{2}(A-B) - BA$$

$$= (A+B)A - (A+B)B - BA$$

$$= A^2 + \cancel{BA} - AB - B^2 \cancel{-BA}$$

$$= \boxed{A^2 - AB - B^2}$$

---

(19) $a = e^{-0} = 1$

$r = e^{-1}$

$n = 100$

$$\left.\right\} \quad S_{100} = \frac{a\left(1-(e^{-1})^{100}\right)}{1-e^{-1}}$$

$$= \boxed{\frac{a\left(1-e^{-100}\right)}{1-e^{-1}}}$$

(20) $S_{12} = 12\,\dfrac{(a_1 + a_{12})}{2} = 156$

$\therefore a_1 + a_{12} = \dfrac{(2)(156)}{12} = 26.$

$\therefore a_2 + a_3 + \cdots + a_{11} = S_{12} - (a_1 + a_{12})$

$= 156 - 26$

$= \boxed{130}$