

Question 1:

(a) [5 points] Solve the following for $0 \leq \theta < 2\pi$: $4 \sin^2 \theta = -3 - 8 \sin \theta$.

$$4 \sin^2 \theta + 8 \sin \theta + 3 = 0$$

$$4 \sin^2 \theta + 2 \sin \theta + 6 \sin \theta + 3 = 0$$

$$2 \sin \theta (2 \sin \theta + 1) + 3 (2 \sin \theta + 1) = 0$$

$$(2 \sin \theta + 1) (2 \sin \theta + 3) = 0$$

$$\therefore 2 \sin \theta + 1 = 0$$

$$\sin \theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$2 \sin \theta + 3 = 0$$

$$\sin \theta = -\frac{3}{2}$$

no solutions.

(b) [5 points] Solve the following for $0 \leq \theta < 2\pi$: $\sin(2\theta) \sin \theta = \cos \theta$.

$$\sin(2\theta) \sin \theta = \cos \theta$$

$$2 \sin \theta \cos \theta \sin \theta = \cos \theta$$

$$[2 \sin^2 \theta - 1] \cos \theta = 0$$

$$2 \sin^2 \theta - 1 = 0$$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

$$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

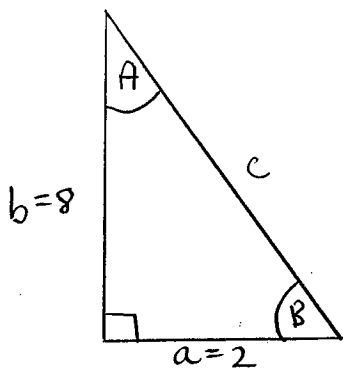
$$\cos \theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}$$

Question 2:

- (a) [4 points] A right angle triangle has $C = 90^\circ$, $a = 2$ and $b = 8$. Determine the values of the remaining angles A and B , and the length of the remaining side c . Round all final answers to two decimal places.

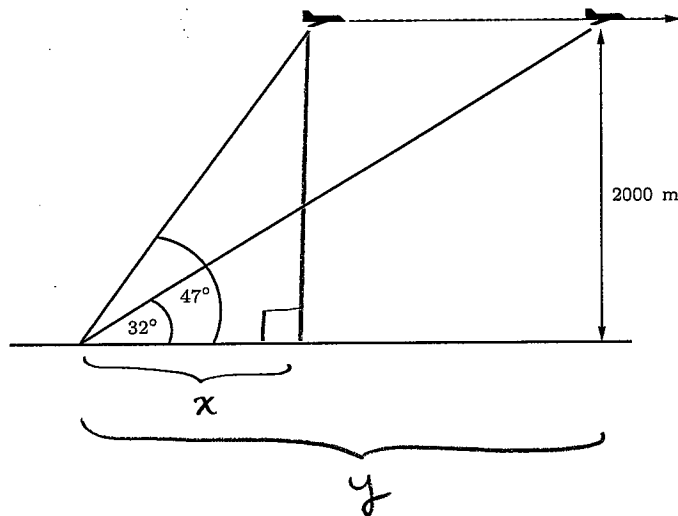


$$c = \sqrt{a^2 + b^2} = \sqrt{2^2 + 8^2} \approx \boxed{8.25}$$

$$A = \tan^{-1}\left(\frac{2}{8}\right) \approx \boxed{14.04}$$

$$B = \tan^{-1}\left(\frac{8}{2}\right) \approx \boxed{75.96}$$

- (b) [6 points] Two planes flying along the same straight line at an altitude of 2000 m pass directly over an observer. A few moments later, the observer on the ground measures the angle of elevation to the nearer plane to be 47° , while that to the more distant plane is 32° . How far apart are the planes when the angle measurements are made? Round your final answer to one decimal place.



$$\tan(32^\circ) = \frac{2000}{y} \Rightarrow y = \frac{2000}{\tan(32^\circ)}$$

$$\tan(47^\circ) = \frac{2000}{x} \Rightarrow x = \frac{2000}{\tan(47^\circ)}$$

$$\therefore \text{distance between planes is } y - x = \frac{2000}{\tan(32^\circ)} - \frac{2000}{\tan(47^\circ)}$$

$$\approx \boxed{1335.6 \text{ m}}$$

Question 3:

(a)[7 points] There are two triangles for which $b = 4$, $c = 5$ and $B = 40^\circ$: determine the remaining sides and angles for each. Round all final answers to two decimal places.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

Case 1:

$$\therefore C = \sin^{-1} \left[\frac{c \sin B}{b} \right]$$

$$= \sin^{-1} \left[\frac{5 \sin(40^\circ)}{4} \right]$$

$$\approx 53.4641^\circ$$

$$\therefore A = 180^\circ - B - C$$

$$= 180^\circ - 40^\circ - 53.4641$$

$$\approx 86.5359^\circ$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\therefore a = \frac{b \sin A}{\sin B} = \frac{4 \sin(86.5359^\circ)}{\sin(40^\circ)} \approx 6.2115$$

$$\therefore a \approx 6.21, A \approx 86.54^\circ, C \approx 53.46^\circ$$

Case 2:

$$C \approx 180^\circ - 53.4641$$

$$= 126.5359$$

$$A \approx 180^\circ - B - C$$

$$= 180^\circ - 40^\circ - 126.5359^\circ$$

$$= 13.4641^\circ$$

$$a = \frac{b \sin A}{\sin B} = \frac{4 \sin(13.4641)}{\sin(40^\circ)}$$

$$\approx 1.4489$$

$$\therefore a \approx 1.45$$

$$A \approx 13.46^\circ$$

$$C \approx 126.54^\circ$$

(b)[3 points] Determine the number of possible triangles for which $a = 2$, $c = 1$ and $C = 100^\circ$. The answer is either 0, 1 or 2; it is not necessary to solve the resulting triangle(s).

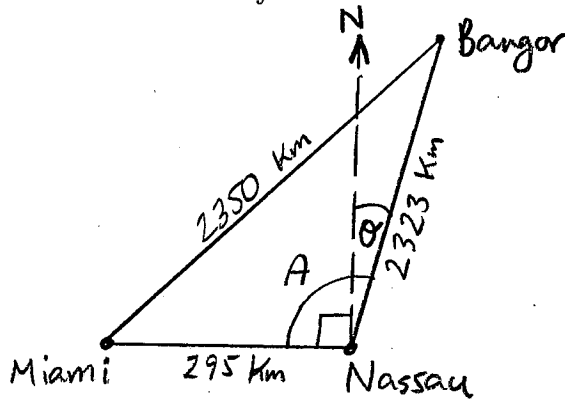
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\therefore \sin A = \frac{a \sin C}{c} = \frac{2 \sin(100^\circ)}{1} \approx 1.9696$$

$\sin A = 1.9696$ has no solution since $-1 \leq \sin A \leq 1$,
so there is no triangle for which $a = 2$, $c = 1$ and $C = 100^\circ$.

Question 4:

- (a) [5 points] Nassau (Bahamas) is 295 km due east of Miami. The city of Bangor (Maine) is northeast of both cities. The distance from Nassau to Bangor is 2323 km, while Miami to Bangor is 2350 km. What bearing would a plane need to take to fly from Nassau to Bangor? Round your final answer one decimal places.



$$2350^2 = 295^2 + 2323^2 - (2)(295)(2323) \cos(A)$$

$$\therefore A = \cos^{-1} \left[\frac{295^2 + 2323^2 - 2350^2}{(2)(295)(2323)} \right]$$

$$\approx 91.6367$$

$$\therefore \theta = A - 90 \approx 1.6$$

\therefore Bearing should be $N 1.6^\circ E$.

- (b) [5 points] A triangle has sides $a = 8$, $b = 5$ and $c = 4$. Determine the sizes of the angles A , B and C . Round all final answers to two decimal places.

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$\therefore C = \cos^{-1} \left[\frac{a^2 + b^2 - c^2}{2ab} \right]$$

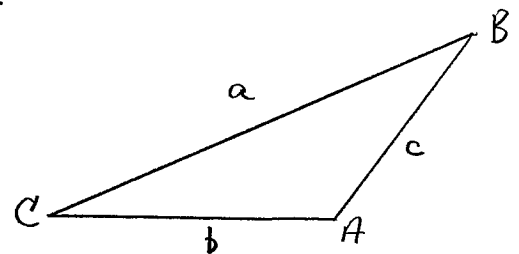
$$= \cos^{-1} \left[\frac{8^2 + 5^2 - 4^2}{(2)(8)(5)} \right]$$

$$\approx 24.1468^\circ$$

$$A = \cos^{-1} \left[\frac{b^2 + c^2 - a^2}{2bc} \right] = \cos^{-1} \left[\frac{5^2 + 4^2 - 8^2}{(2)(5)(4)} \right] \approx 125.0996$$

$$B = \cos^{-1} \left[\frac{a^2 + c^2 - b^2}{(2)ac} \right] = \cos^{-1} \left[\frac{8^2 + 4^2 - 5^2}{(2)(8)(4)} \right] \approx 30.7535$$

$$\therefore A \approx 125.10^\circ, B \approx 30.75^\circ, C \approx 24.15^\circ$$



Question 5 [10 points]: Solve the following system of equations using matrix reduction (no credit will be given for using any other method). Clearly state the row operations used at each step and clearly state the solution set.

$$-2x + 5y - 19z = 1$$

$$x - 2y + 7z = 1$$

$$3x - 3y + 9z = 6$$

$$\left[\begin{array}{ccc|c} -2 & 5 & -19 & 1 \\ 1 & -2 & 7 & 1 \\ 3 & -3 & 9 & 6 \end{array} \right]$$

$$r_1 \leftrightarrow r_2: \left[\begin{array}{ccc|c} 1 & -2 & 7 & 1 \\ -2 & 5 & -19 & 1 \\ 3 & -3 & 9 & 6 \end{array} \right]$$

$$\begin{array}{l} R_2 = 2r_1 + r_2: \\ R_3 = -3r_1 + r_3: \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 7 & 1 \\ 0 & 1 & -5 & 3 \\ 0 & 3 & -12 & 3 \end{array} \right]$$

$$R_3 = -3r_2 + r_3: \left[\begin{array}{ccc|c} 1 & -2 & 7 & 1 \\ 0 & 1 & -5 & 3 \\ 0 & 0 & 3 & -6 \end{array} \right]$$

$$R_3 = \frac{1}{3}r_3: \left[\begin{array}{ccc|c} 1 & -2 & 7 & 1 \\ 0 & 1 & -5 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\therefore z = -2$$

$$y = 3 + 5z = 3 + 5(-2) = -7$$

$$\begin{aligned} x &= 1 + 2y - 7z \\ &= 1 + 2(-7) - 7(-2) \\ &= 1 - 14 + 14 \\ &= 1 \end{aligned}$$

$$\therefore x=1, y=-7, z=-2,$$