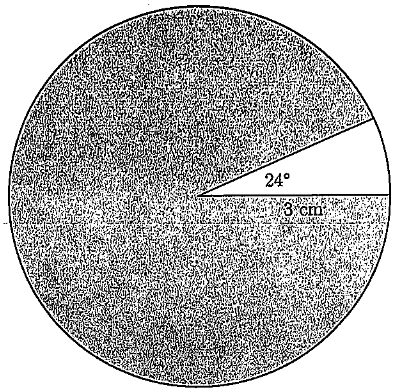
Question 1:

(a)[4 points] Determine the area of the shaded region in the following figure (round your answer to 2 decimals):



$\theta = 360^\circ - 24^\circ = 336^\circ$

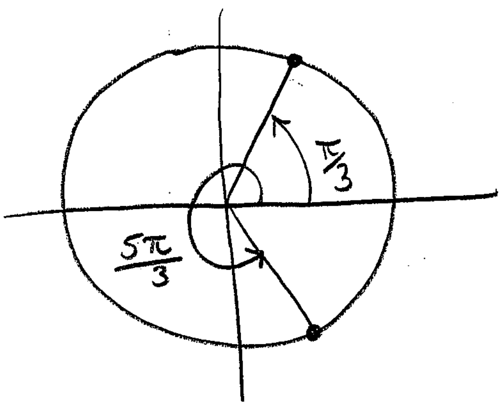
$(336^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{336\pi}{180}$ radians.

$\therefore$ area $A = \frac{1}{2}\theta r^2$

$= \left(\frac{1}{2}\right)\left(\frac{336\pi}{180}\right)(3^2)$

$\approx \boxed{26.39 \text{ cm}^2}$

(b)[3 points] Simplify $\cos^{-1}\left[\cos\left(\frac{5\pi}{3}\right)\right]$.



$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$

$\therefore \cos^{-1}\left[\cos\left(\frac{5\pi}{3}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$

$= \boxed{\frac{\pi}{3}}$

(c)[3 points] Given that $\cos\left(\frac{\pi}{16}\right) = b$, find an expression involving b for $\cos\left(-\frac{65\pi}{16}\right)$.

$\cos\left(-\frac{65\pi}{16}\right) = \cos\left(\frac{65\pi}{16}\right)$ since cosine is even

$= \cos\left(\frac{64\pi + \pi}{16}\right)$

$= \cos\left(4\pi + \frac{\pi}{16}\right)$

$= \cos\left(\frac{\pi}{16}\right)$ by periodicity

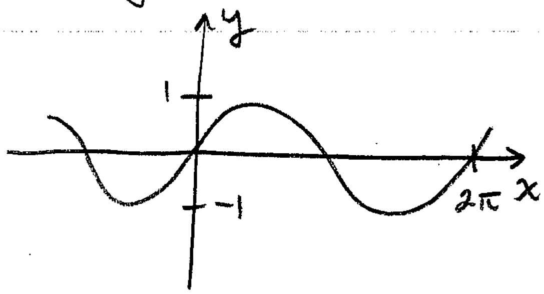
$= \boxed{b}$

Question 2:

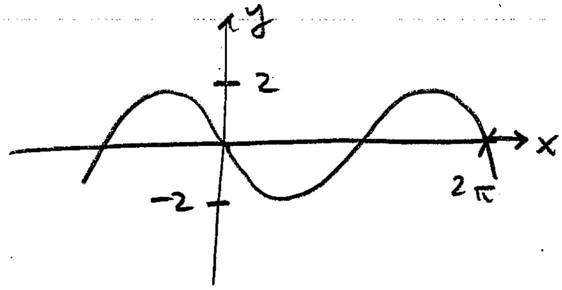
(a)[**7 points**] Carefully (and neatly!) sketch the graph of $y = 2\sin(-3x - \pi) - 1$. Show at least one complete period, label your graph and indicate the scale on the axes.

$$y = 2\sin(-3x-\pi)-1 = 2\sin\left(-3\left(x+\frac{\pi}{3}\right)\right)-1 = -2\sin\left(3\left(x+\frac{\pi}{3}\right)\right)-1$$

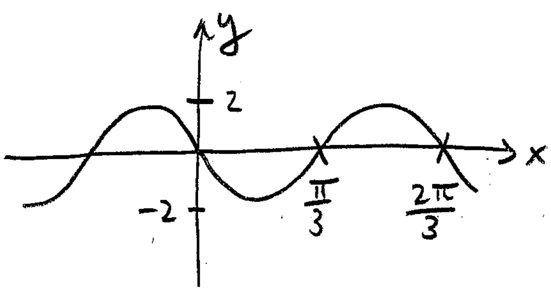
① $y = \sin(x)$



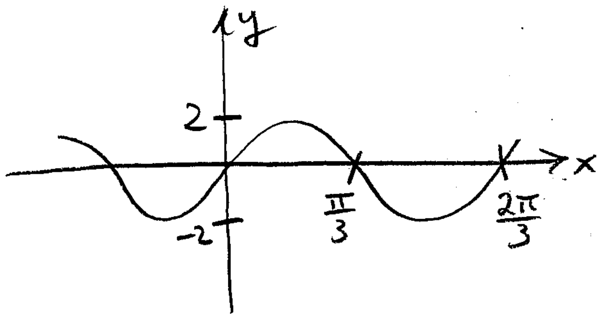
② $y = -2\sin(x)$



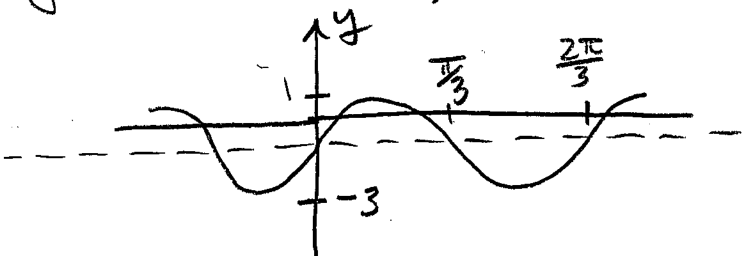
③ $y = -2\sin(3x)$



④ $y = -2\sin\left(3\left(x+\frac{\pi}{3}\right)\right)$



⑤ $y = -2\sin\left(3\left(x+\frac{\pi}{3}\right)\right)-1$



(b)[**3 points**] State the period, amplitude and phase shift of the function graphed in part (a).

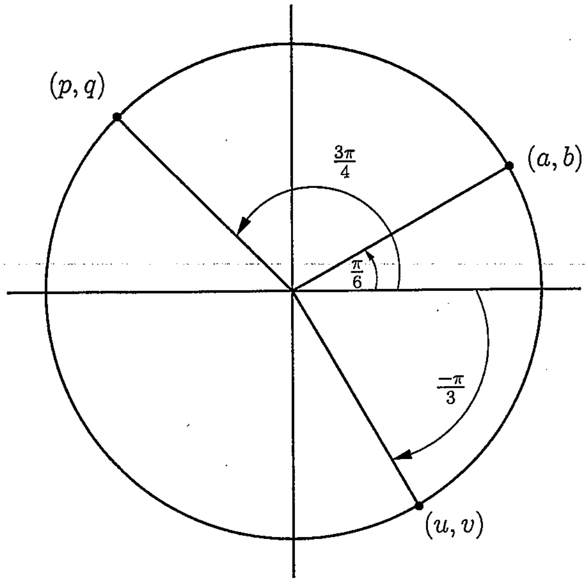
period $T = \frac{2\pi}{3}$

amplitude $= |-2| = 2$.

phase-shift $= -\frac{\pi}{3}$.

Question 3:

(a) [**6 points**] Determine the exact value of a, b, p, q, u and v in the following unit circle:



$a = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$b = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$p = \cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$q = \sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$u = \cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$v = \sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

(b) [**2 points**] If $\sin\theta = -5/13$ where θ is in the fourth quadrant, what is $\cot\theta$.

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{5}{13}\right)^2} = \sqrt{\frac{169-25}{169}} = \frac{12}{13}$

$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\left(\frac{12}{13}\right)}{\left(\frac{-5}{13}\right)} = \boxed{\frac{-12}{5}}$.

(c) [**2 points**] Convert $\frac{7\pi}{5}$ to degrees.

$\left(\frac{7\pi}{5}\right)\left(\frac{180^\circ}{\pi \text{ rad}}\right) = \boxed{252^\circ}$

Question 4:

(a) **[4 points]** Simplify $\sec(\sin^{-1} x)$. Your answer should not contain any trigonometric or inverse trigonometric function.

Let $\theta = \sin^{-1} x$, so $\sin\theta = x$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

We want $\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$.

$\therefore \sec\theta = \boxed{\frac{1}{\sqrt{1-x^2}}}$

(b) **[4 points]** Determine the exact value of $\tan\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]$.

Let $\theta = \cos^{-1}\left(-\frac{3}{5}\right)$, so $\cos\theta = \frac{-3}{5}$, $0 \le \theta \le \pi$

$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{-3}{5}\right)^2} = \sqrt{\frac{25-9}{25}} = \frac{4}{5}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{4}{5}\right)}{\left(\frac{-3}{5}\right)} = \boxed{\frac{-4}{3}}$

(c) **[2 points]** Determine the exact value of $\sin\left(\frac{5\pi}{12}\right)$.

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$$
$$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$
$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$= \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

Question 5:

(a) [4 points] Given that $\sin\left[\cos^{-1}\left(-\frac{4}{5}\right)\right] = \frac{3}{5}$, determine the exact value of

$$\cos\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{4}{5}\right)\right].$$

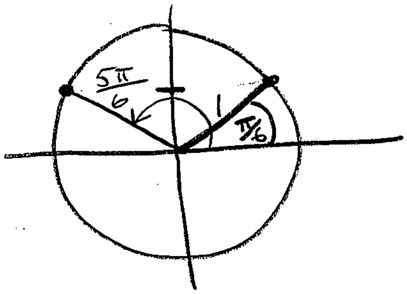
$$\cos\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{4}{5}\right)\right] = \cos\left[-\frac{\pi}{4} + \cos^{-1}\left(-\frac{4}{5}\right)\right]$$

$$= \cos\left(-\frac{\pi}{4}\right)\cos\left(\cos^{-1}\left(-\frac{4}{5}\right)\right) - \sin\left(-\frac{\pi}{4}\right)\sin\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{4}{5}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{3}{5}\right)$$

$$= \boxed{\frac{-1}{5\sqrt{2}}}$$

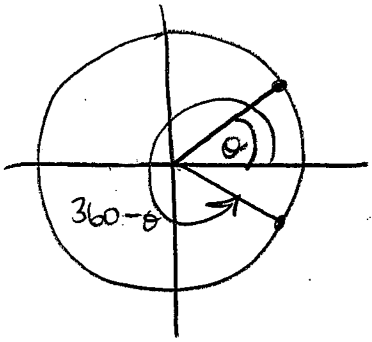
(b) [3 points] Find all angles $0 \le \theta < 2\pi$ for which $\sin\theta = 1/2$.



$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}.$$

(c) [3 points] Determine the exact value of $\sin(1°) + \sin(2°) + \sin(3°) + \cdots + \sin(359°)$. (Hint: think of the unit circle)

For $0 \le \theta < 180°$, $\sin\theta + \sin(360° - \theta) = 0$



$\therefore \sin(1°) + \sin(2°) + \cdots + \sin(359°)$

$$= \left[\underbrace{\sin(1°) + \sin(359°)}_{\to 0}\right] + \left[\underbrace{\sin(2°) + \sin(358°)}_{\to 0}\right]$$

$$+ \cdots + \left[\underbrace{\sin(179°) + \sin(181°)}_{\to 0}\right] + \underbrace{\sin(180°)}_{\to 0}$$

$$= \boxed{0}.$$