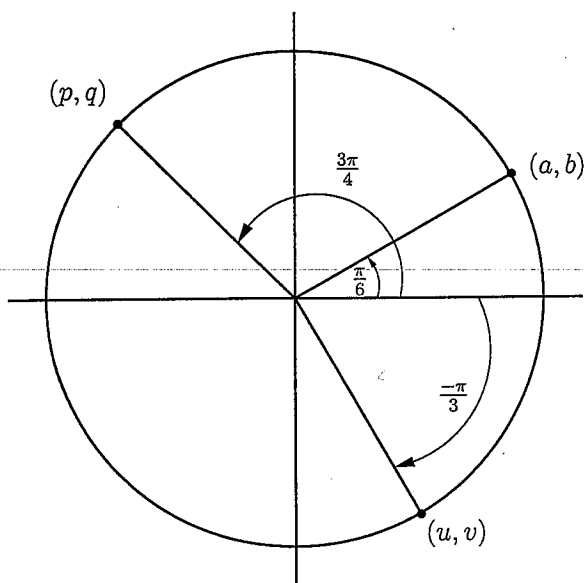**Question 1:**

(a)[6 points] Determine the exact value of a, b, p, q, u and v in the following unit circle:

$a = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$b = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$p = \cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$q = \sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$u = \cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$v = \sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

(b)[2 points] Convert $\dfrac{7\pi}{5}$ to degrees.

$\left(\frac{7\pi}{5}\right)\left(\frac{180°}{\pi \; rad}\right) = \boxed{252°}$

(c)[2 points] If $\sin\theta = -5/13$ where θ is in the fourth quadrant, what is $\cot\theta$.

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{-5}{13}\right)^2} = \sqrt{\frac{169-25}{169}} = \frac{12}{13}$

$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\left(\frac{12}{13}\right)}{\left(\frac{-5}{13}\right)} = \boxed{\frac{-12}{5}}$
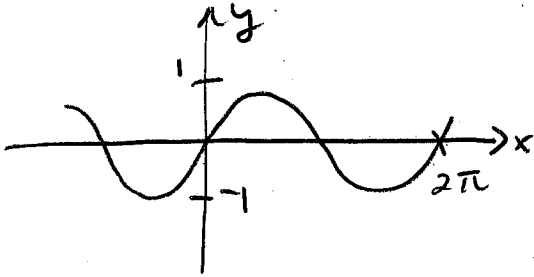
Question 2:

(a)[7 points] Carefully (and neatly!) sketch the graph of $y = -2\sin(-3x + \pi) + 1$. Show at least one complete period, label your graph and indicate the scale on the axes.
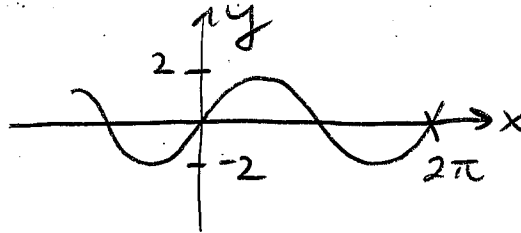
$$y = -2\sin(-3x+\pi)+1 = -2\sin\left(-3\left(x-\tfrac{\pi}{3}\right)\right)+1$$

$$= 2\sin\left(3\left(x-\tfrac{\pi}{3}\right)\right)+1$$
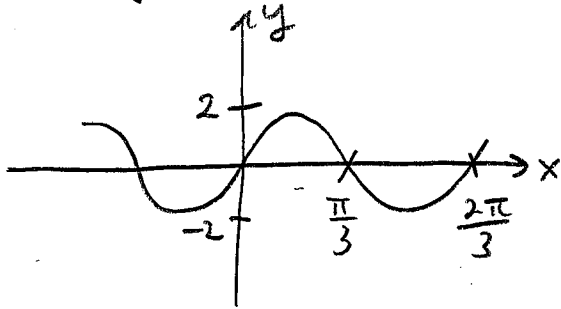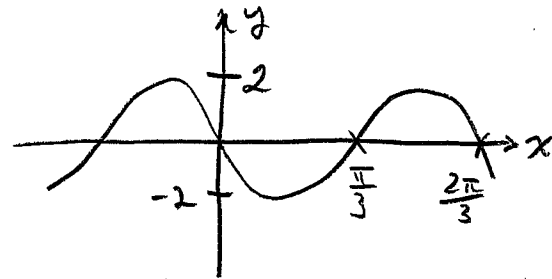
① $y = \sin(x)$
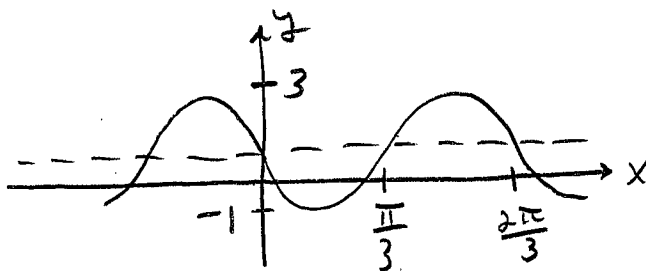

② $y = 2\sin(x)$

③ $y = 2\sin(3x)$

④ $y = 2\sin\left(3\left(x - \tfrac{\pi}{3}\right)\right)$

⑤

(b)[3 points] State the period, amplitude and phase shift of the function graphed in part (a).
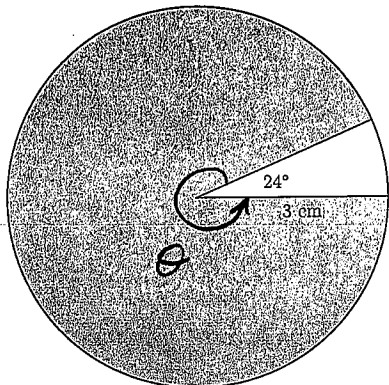
Period $\quad T = \dfrac{2\pi}{3}$

amplitude $= |2| = 2$.

phase - shift $= \dfrac{\pi}{3}$

Question 3:

(a)[4 points] Determine the area of the shaded region in the following figure (round your answer to 2 decimals):

$$\theta = 360° - 24° = 336°$$

$$(336°)\left(\frac{\pi}{180°}\right) = \frac{336\pi}{180} \text{ radians.}$$

$$\therefore \text{ area } A = \frac{1}{2}\theta r^2$$

$$= \left(\frac{1}{2}\right)\left(\frac{336\pi}{180}\right)\left(3^2\right)$$

$$\approx \boxed{26.39 \text{ cm}^2}$$

(b)[3 points] Given that $\cos\left(\frac{\pi}{16}\right) = b$, find an expression involving b for $\cos\left(-\frac{65\pi}{16}\right)$.

$$\cos\left(-\frac{65\pi}{16}\right) = \cos\left(\frac{65\pi}{16}\right) \text{ since cosine is even}$$

$$= \cos\left(\frac{64\pi + \pi}{16}\right)$$

$$= \cos\left(4\pi + \frac{\pi}{16}\right)$$

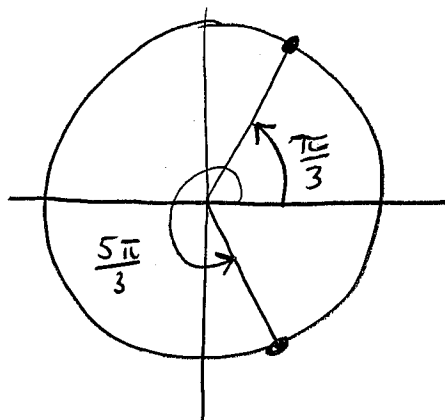$$= \cos\left(\frac{\pi}{16}\right) \text{ by periodicity}$$

$$= \boxed{b}$$

(c)[3 points] Simplify $\cos^{-1}\left[\cos\left(\frac{5\pi}{3}\right)\right]$.

$$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore \cos^{-1}\left[\cos\left(\frac{5\pi}{3}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$$

$$= \boxed{\frac{\pi}{3}}$$

Question 4:

(a)[4 points]   Determine the exact value of $\tan\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]$.

Let $\theta = \cos^{-1}\left(-\frac{3}{5}\right)$, so $\cos(\theta) = \frac{-3}{5}$, $0 \le \theta \le \pi$

$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\frac{3}{5}\right)^2} = \sqrt{\frac{25-9}{25}} = \frac{4}{5}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)} = \boxed{\frac{-4}{3}}$.

(b)[4 points]   Simplify $\sec\left(\sin^{-1}x\right)$. Your answer should not contain any trigonometric or inverse trigonometric function.

Let $\theta = \sin^{-1}x$, so $\sin\theta = x$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

We want $\sec\theta = \frac{1}{\cos\theta}$.

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$

$\therefore \sec\theta = \boxed{\frac{1}{\sqrt{1-x^2}}}$

(c)[2 points]   Determine the exact value of $\sin\left(\frac{5\pi}{12}\right)$.

$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

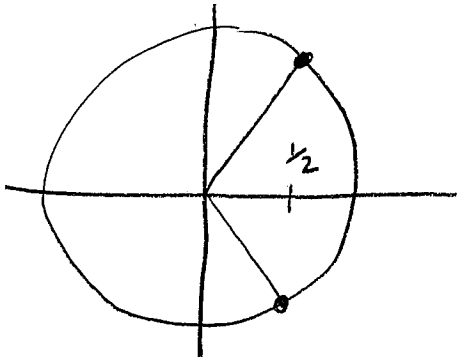$= \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

Question 5:

(a)[4 points]   Given that $\sin\left[\cos^{-1}\left(-\dfrac{4}{5}\right)\right] = \dfrac{3}{5}$, determine the exact value of

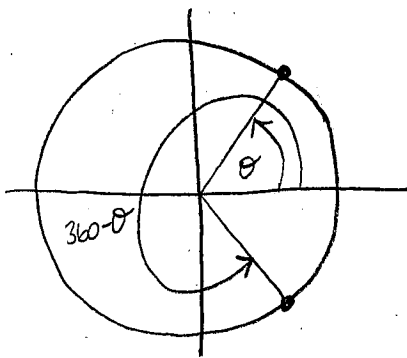$$\cos\left[\tan^{-1}(-1) + \cos^{-1}\left(-\dfrac{4}{5}\right)\right].$$

$$\cos\left[\tan^{-1}(-1) + \cos^{-1}\left(-\tfrac{4}{5}\right)\right] = \cos\left[-\tfrac{\pi}{4} + \cos^{-1}\left(-\tfrac{4}{5}\right)\right]$$

$$= \cos\left(-\tfrac{\pi}{4}\right)\cos\left[\cos^{-1}\left(-\tfrac{4}{5}\right)\right] - \sin\left(-\tfrac{\pi}{4}\right)\sin\left[\cos^{-1}\left(-\tfrac{4}{5}\right)\right]$$

$$= \left(\tfrac{1}{\sqrt{2}}\right)\left(-\tfrac{4}{5}\right) + \left(\tfrac{1}{\sqrt{2}}\right)\left(\tfrac{3}{5}\right)$$

$$= \boxed{\dfrac{-1}{5\sqrt{2}}}$$

(b)[3 points]   Find all angles $0 \le \theta < 2\pi$ for which $\cos\theta = 1/2$.

$$\theta = \dfrac{\pi}{3} \, , \, \dfrac{5\pi}{3}.$$

(c)[3 points]   Determine the exact value of $\sin(1°) + \sin(2°) + \sin(3°) + \cdots + \sin(359°)$. (Hint: think of the unit circle)

For $0 \le \theta < 180°$, $\sin\theta + \sin(360° - \theta) = 0$

$$\therefore \sin(1°) + \sin(2°) + \cdots + \sin(359°)$$

$$= \underbrace{\left[\sin(1°) + \sin(359°)\right]}_{0} + \underbrace{\left[\sin(2°) + \sin(358°)\right]}_{0}$$

$$+ \cdots + \underbrace{\left[\sin(179°) + \sin(181°)\right]}_{0} + \underbrace{\sin(180°)}_{0}$$

$$= 0.$$