Question 1:

(a) [**4 points**] Let $f(x) = \sqrt{x-2}$ and $g(x) = 1-2x$. Determine and simplify $f \circ g$. State the domain.

$$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)-2} = \underbrace{\sqrt{(1-2x)-2}}_{*} = \sqrt{-2x-1}$$

using $*$, must have $(1-2x)-2 \geq 0$

$$-1 \geq 2x$$

$$\therefore \; x \leq \frac{-1}{2}$$

$\therefore$ domain of $f \circ g$ is $(-\infty, -\frac{1}{2}]$.

(b) [**4 points**] Again let $f(x) = \sqrt{x-2}$ and $g(x) = 1-2x$. Determine and simplify $g \circ f$. State the domain.

$$(g \circ f)(x) = g(f(x)) = 1 - 2f(x) = \underbrace{1-2\sqrt{x-2}}_{*}$$

using $*$, must have $x-2 \geq 0$,

$$x \geq 2$$

$\therefore$ domain of $g \circ f$ is $[2, \infty)$.

(c) [**2 points**] Let $H(x) = \dfrac{\sqrt{x+1}}{2}$ and $g(x) = x+1$. Determine the function f so that $H(x) = (f \circ g)(x)$.

$$H(x) = \frac{\sqrt{x+1}}{2} = \frac{\sqrt{g(x)}}{2}$$

$$\therefore \; f(x) = \frac{\sqrt{x}}{2}.$$

Question 2:

(a)[**5 points**] Let $f(x) = \dfrac{2x-3}{x+4}$. Determine $f^{-1}(x)$.

$$y = \frac{2x-3}{x+4}$$

$\underline{x \leftrightarrow y}$: $x = \dfrac{2y-3}{y+4}$

$$xy + 4x = 2y - 3$$
$$xy - 2y = -3 - 4x$$
$$y(x-2) = -3-4x$$
$$y = \frac{-3-4x}{x-2}$$
$$\therefore f^{-1}(x) = \frac{-3-4x}{x-2}, \quad x \neq 2$$

(b)[**3 points**] Use part (a) to determine the domain and range of $f(x) = \dfrac{2x-3}{x+4}$.

$f(x) = \dfrac{2x-3}{x+4}$ has domain $\{x \in \mathbb{R} \mid x \neq -4\}$

$\underline{\text{or}}$ $(-\infty, -4) \cup (-4, \infty)$.

The range of $f(x)$ is the domain of $f^{-1}(x)$ which is $\{x \in \mathbb{R} \mid x \neq 2\}$, or $(-\infty, 2) \cup (2, \infty)$.

(c)[**2 points**] If $(3, -7)$ and $(5, 3)$ are points on the graph of a function $g(x)$, calculate the value of $g^{-1}(3) - g(3)$.

$(3,-7)$ a point on graph of $y = g(x) \Rightarrow g(3) = -7$ and $g^{-1}(-7) = 3$.

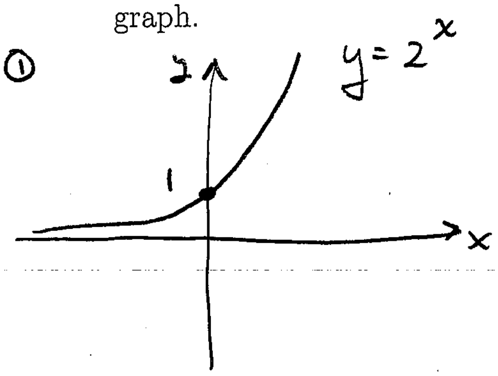
$(5,3)$ a point on graph of $y = g(x) \Rightarrow g(5) = 3$ and $g^{-1}(3) = 5$.

$\therefore g^{-1}(3) - g(3) = 5 - (-7) = 12$.

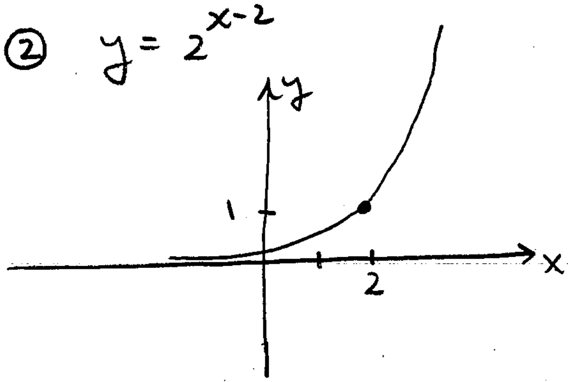
Question 3:

(a) [**5 points**] Neatly sketch the graph of $f(x) = 2^{x-2} - 1$. Indicate at least one point on your graph.

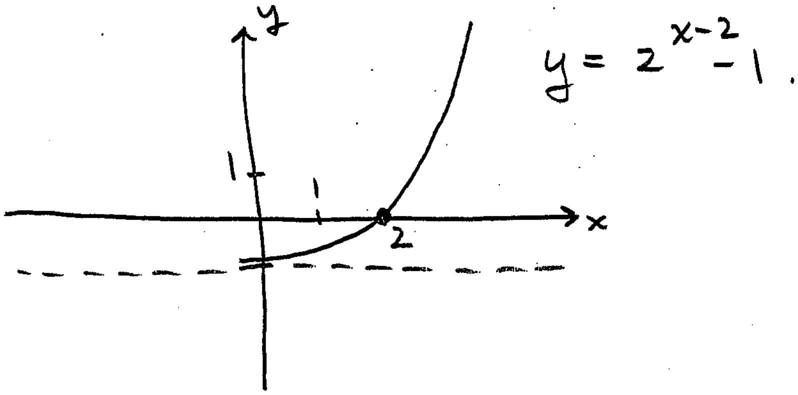
1



2



3



(b) [**5 points**] Solve for x:

$$2^{x^2+7x} = 4^{x+7}$$

$$2^{x^2+7x} = (2^2)^{x+7}$$

$$2^{x^2+7x} = 2^{2x+14}$$

$$x^2 + 7x = 2x + 14$$

$$x^2 + 5x - 14 = 0$$

$$(x-2)(x+7) = 0$$

$$x = 2 \quad , \quad x = -7$$

Question 4:

(a) [**2 points**] Compute a decimal approximation to $\log_7 19$. (Round your final answer to 3 decimal places.)

$$\log_7 19 = \frac{\ln 19}{\ln 7} \approx 1.513$$

(b) [**4 points**] Express the following as sums and/or differences of logarithms. Where possible express powers as factors:

$$\ln\left[\frac{(x-4)^2}{x^2-1}\right]^{2/3}$$

$$= \frac{2}{3}\ln\left[\frac{(x-4)^2}{x^2-1}\right]$$

$$= \frac{2}{3}\ln(x-4)^2 - \frac{2}{3}\ln(x^2-1)$$

$$= \frac{4}{3}\ln(x-4) - \frac{2}{3}\ln\left[(x-1)(x+1)\right]$$

$$= \frac{4}{3}\ln(x-4) - \frac{2}{3}\ln(x-1) - \frac{2}{3}\ln(x+1).$$

(c) [**4 points**] Write as a single logarithm:

$$8\log_2\sqrt{3x-2} - \log_2\left(\frac{4}{x}\right) + \log_2 4$$

$$= \log_2\left(\sqrt{3x-2}\right)^8 \cancel{- \log_2 4} + \log_2 x \cancel{+ \log_2 4}$$

$$= \log_2(3x-2)^4 + \log_2 x$$

$$= \log_2\left[x(3x-2)^4\right].$$

Question 5:

(a) [5 points] Solve the following logarithmic equation for x:

$$\log_6(x+4) + \log_6(x+3) = 1$$

note: $x+4>0$, $x+3>0$
$x>-4$, $x>-3$
$\therefore x>-3$

$$\log_6[(x+4)(x+3)] = 1$$
$$(x+4)(x+3) = 6$$
$$x^2+7x+6 = 0$$
$$(x+6)(x+1) = 0$$
$$\cancel{x=-6}, \quad x=-1$$

since $x>-3$

$$\therefore \quad x = -1$$

(b) [5 points] A certain population grows according to the model $P(t) = 500e^{kt}$ where t represents time in days and $t = 0$ corresponds to the present. If the population doubles in five days, how long will it take to grow from 500 to 2500 individuals? Round your answer to the nearest day.

$P(5) = 2P(0)$, so $\quad 500e^{k\cdot 5} = 1000$
$$e^{5k} = 2$$
$$5k = \ln 2$$
$$k = \frac{\ln 2}{5}$$

Now solve $\quad 500e^{\left(\frac{\ln 2}{5}\right)t} = 2500$ for t
$$e^{\left(\frac{\ln 2}{5}\right)t} = 5$$
$$\left(\frac{\ln 2}{5}\right)t = \ln 5$$
$$\therefore t = \frac{\ln 5}{\left(\frac{\ln 2}{5}\right)} \approx 12 \text{ days}$$