

(1) [7 points] Let $f(x) = \frac{3}{x-1}$ and $g(x) = \frac{2}{x}$.

(i) Determine and simplify $f \circ g$.

(ii) Determine the domain of $f \circ g$.

$$\begin{aligned} \text{(i)} \quad (f \circ g)(x) &= f(g(x)) \\ &= \frac{3}{\frac{2}{x} - 1} \quad \left. \vphantom{\frac{3}{\frac{2}{x} - 1}} \right\} \dots (*) \\ &= \frac{3}{\frac{2-x}{x}} \\ &= \frac{3x}{2-x} \end{aligned}$$

(ii) Using (*), $x \neq 2$, $x \neq 0$,

\therefore Domain of $f \circ g$ is $\{x \in \mathbb{R} \mid x \neq 0, x \neq 2\}$.

(2) [8 points] Let $f(x) = \frac{2x}{3x-1}$. Determine $f^{-1}(x)$ and use your result to find both the domain and range of $f(x)$.

$$f(x) = \frac{2x}{3x-1}, \quad \text{domain of } f \text{ is } \left\{ x \in \mathbb{R} \mid x \neq \frac{1}{3} \right\}$$

$$y = \frac{2x}{3x-1}$$

$$x = \frac{2y}{3y-1}$$

$$3xy - x = 2y$$

$$3xy - 2y = x$$

$$y(3x-2) = x$$

$$y = \frac{x}{3x-2}$$

$$\therefore f^{-1}(x) = \frac{x}{3x-2}$$

, domain is $\left\{ x \in \mathbb{R} \mid x \neq \frac{2}{3} \right\}$.

\therefore range of f is $\left\{ y \in \mathbb{R} \mid y \neq \frac{2}{3} \right\}$.