Question 1 [10 points]:

(a) [**5**] Determine $f^{-1}(x)$ if $f(x) = \dfrac{3x+1}{2x-1}$.

(b) [**3**] Express as a single logarithm: $\log_5(x+3) - \log_5(3x+1) - 3\log_5 2$

(c) [**2**] Calculate $\log_\pi 2$ (round your answer to two decimal places).

Question 2 [10 points]: Solve the following equations for x:

(a) [5] $\dfrac{e^{x^2}}{e^6} = e^x$

(b) [5] $\log_2(x+2) + \log_2(x-1) = 2$

Question 3 [10 points]:

(a) [5] A 10 g sample of radioactive material reduces to 6 g after 25 days. What is the half life of the material? Round your answer to two decimal places. (Recall that the amount A of radioactive material present at time t is given by $A(t) = A_0 e^{kt}$ where A_0 is the original amount of radioactive material and k is a negative constant.)

(b) [5] A population grows according to the model $P(t) = P_0 e^{kt}$, where k is a positive constant, t is time in years, and $t = 0$ corresponds to the present. If the population in five years time is 5000 and reaches 7000 after a further two years, what is the current population size? Round your answer to two decimal places.

Question 4 [10 points]:

(a) [2] Convert $31\pi/9$ to degrees.

(b) [3] Determine the exact value of $\sin\left(\frac{5\pi}{6}\right)\sec\left(\frac{5\pi}{4}\right)$.

(d) [4] If $\cos\theta = 3/5$ determine all possible values of $\sin\theta$.

Question 5 [10 points]:

(a) [7] Carefully sketch the graph of $y = 2\cos(4x - \pi) + 1$.

(b) [3] State the period, amplitude and phase-shift of the function sketched in part (a).

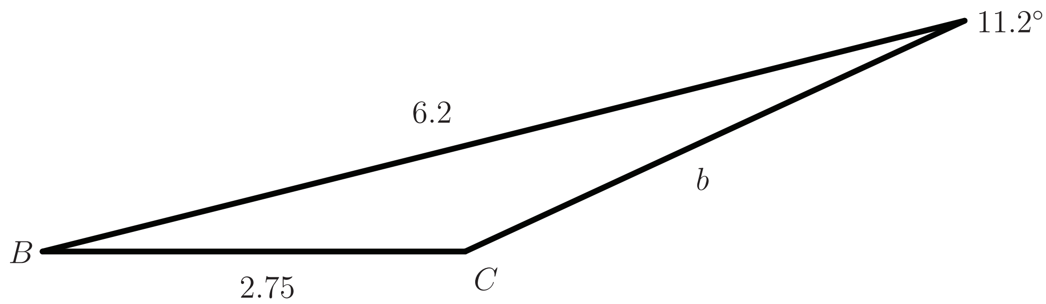
Question 6 [10 points]: Find all solutions in the interval $[0, 2\pi)$:

(a) [5] $2\cos^2\theta - 3\cos\theta = 2$

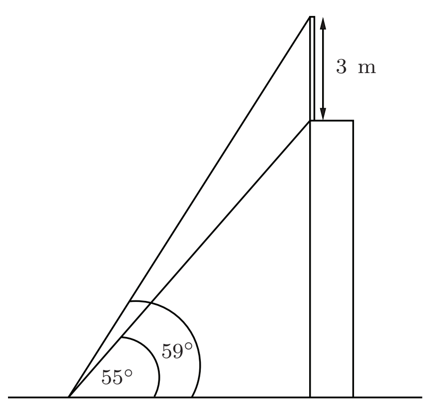
(b) [5] $\cos(2\theta) = \sin(\theta)$

Question 7 [10 points]:

(a) [5] Solve for all missing sides and angles in the following triangle. The figure is to scale. Round final answers to one decimal place.



(b) [5] An observer on the ground is looking up at a 3 m tall antenna mounted atop a building. The angle of elevation to the bottom of the antenna is 55°, while that to the top of the antenna is 59°. How far is the person from the the building? Round your final answer to one decimal place.



Question 8 [10 points]: Let

$$\mathbf{A} = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix}$$

(a) [3] Compute $(4A - 3B)C$.

(b) [2] If there is a matrix D such that the product CDA is defined, what must be the size of D?.

(c) [5] Determine A^{-1} .

Question 9 [10 points]: Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method). Clearly state your conclusion:

$$\begin{aligned} x + y + z &= 0 \\ x - 2y + 2z &= 4 \\ x + 2y - z &= 2 \end{aligned}$$

Question 10 [10 points]:

(a) [**2**] Determine a_{11} in the geometric sequence $\pi, -\pi/\sqrt{2}, \pi/2, \ldots$.

(b) [**3**] Compute the sum of the arithmetic series $7 + 11/2 + \cdots + a_{23}$.

(c) [**5**] \$1200 is deposited on January 1 of each year into a fund which pays interest at 6.5% compounded annually. The first deposit is made on January 1 2009. What is the accumulated value of the fund just after the last deposit is made on January 1 2025? Round your answer to the nearest dollar. Recall that P dollars invested at i% interest compounded annually accumulates to an amount $A = P\left(1 + \frac{i}{100}\right)^n$ by the end of n years.