Question 1:

(a)[3 points] Determine y' if $y = 5x^4 - \pi x + 6\sqrt{x} - \sqrt{2}$ $= 5x^4 - \pi x + 6x^{1/2} - \sqrt{2}$

$$y' = 20x^3 - \pi + 3x^{-\frac{1}{2}}$$

(b)[3 points] Determine $\frac{dy}{dx}$ where $y = \left(3x^2 + \frac{1}{x}\right)\csc x$ $= (3x^2 + x^{-1})\csc x$

$$\therefore \frac{dy}{dx} = (6x - x^{-2})\csc x + (3x^2 + x^{-1})(-\csc x \cot x)$$

(c)[4 points] Determine $f'(x)$ if $f(x) = \frac{\sec x - \sqrt[3]{x}}{\tan x - 2x^{1/2}}$ $= \frac{\sec x - x^{\frac{1}{3}}}{\tan x - 2x^{1/2}}$

$$f'(x) = \frac{(\tan x - 2x^{1/2})(\sec x \tan x - \frac{1}{3}x^{-2/3}) - (\sec x - x^{1/3})(\sec^2 x - x^{-1/2})}{(\tan x - 2x^{1/2})^2}$$

Question 2:

(a)[**3 points**] Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x+\sqrt{x}}$ $= (x + x^{1/2})^{1/3}$

$$\frac{dy}{dx} = \frac{1}{3}(x+x^{\frac{1}{2}})^{-2/3}\left(1+\frac{1}{2}x^{-\frac{1}{2}}\right)$$

(b)[**3 points**] Find $\frac{dy}{dx}$ if $y = \sin(\sqrt{7x+\cos x})$ $= \sin\left((7x+\cos x)^{\frac{1}{2}}\right)$

$$\frac{dy}{dx} = \cos\left((7x+\cos x)^{\frac{1}{2}}\right)\cdot\frac{1}{2}(7x+\cos x)^{-\frac{1}{2}}(7-\sin x)$$

(c)[**4 points**] Compute $g''(\pi)$ if $g(\theta) = \theta^2\cos\theta$

$$g'(\theta) = 2\theta\cos\theta - \theta^2\sin\theta$$

$$g''(\theta) = 2\cos\theta - 2\theta\sin\theta - 2\theta\sin\theta - \theta^2\cos\theta$$

$$g''(\pi) = 2\cos\pi - \cancel{2\pi\sin\pi}^{\,0} - \cancel{2\pi\sin\pi}^{\,0} - \pi^2\cos\pi$$

$$= -2 + \pi^2$$

Question 3:

(a)[**5 points**] Determine the values of a and b for which the line $y = -2x + b$ is tangent to the parabola $y = ax^2$ when $x = 2$.

Since the tangent line $y = -2x+b$ and curve $y = ax^2$ share the point of contact,

$$-2(2)+b = a(2)^2$$

$$\therefore 4a - b = -4 \quad \ldots (*)$$

Also, $\frac{d}{dx}\left[ax^2\right]\Big|_{x=2} = -2$, the slope of the tangent line

$$\therefore 2ax\Big|_{x=2} = -2$$

$$\therefore 4a = -2 \Rightarrow a = \frac{-2}{4} = \frac{-1}{2}$$

From $(*)$: $4\left(\frac{-1}{2}\right) - b = -4 \Rightarrow b = -2+4 = 2$

$$\boxed{\therefore \quad a = \frac{-1}{2}, \quad b = 2}$$

(b)[**5 points**] A particle moving along a straight line has position at time t given by $s(t) = 2t^3 - 12t^2 + t + 2$. Here $t \geq 0$ is in seconds and position is in metres. Determine the velocity of the particle when the acceleration is 0 m/s^2.

velocity $v(t) = s'(t) = 6t^2 - 24t + 1$

acceleration $a(t) = s''(t) = 12t - 24$

$a(t) = 0 \Rightarrow 12t - 24 = 0$

$\Rightarrow t = \frac{24}{12} = 2$ s.

$$\therefore v(2) = 6(2)^2 - 24(2) + 1$$

$$= \boxed{-23 \ \frac{m}{s}}$$

Question 4:

(a)[4 points] Determine $f'(x)$ if $f(x) = (1-x^2)\sin\left(\frac{1}{x}\right)$ $= (1-x^2)\sin(x^{-1})$

$$f'(x) = -2x\sin(x^{-1}) + (1-x^2)\cos(x^{-1})(-x^{-2})$$

(b)[6 points] Find an equation of the tangent line to the curve $x\sin(y-xy) = \frac{x-y}{y}$ at the point $(1,1)$.

$$\frac{d}{dx}\left[x\sin(y-xy)\right] = \frac{d}{dx}\left[\frac{x-y}{y}\right]$$

$$1\cdot\sin(y-xy) + x\cos(y-xy)(y'-y-xy') = \frac{(y)(1-y')-(x-y)y'}{y^2}$$

at $(1,1)$:

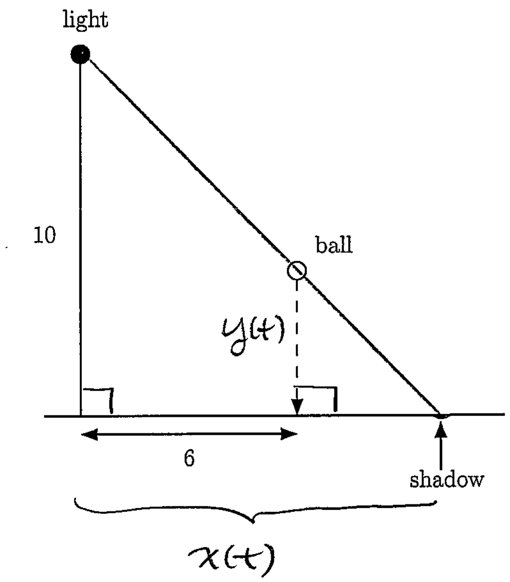
$$1\cdot\cancel{\sin(0)}^{\,0} + 1\cdot\cos(0)(\cancel{y'}-1-\cancel{y'}) = \frac{(1)(1-y')-\cancel{(1-1)y'}^{\,0}}{(1)^2}$$

$$-1 = 1-y'$$

$$\therefore\ y' = 2$$

$$\boxed{\therefore\ y-1 = 2(x-1)}$$

Question 5 [10 points]: A ball is falling vertically to the ground near a 10 m tall lamp post. It is night time, so the light atop the lamp post casts the ball's shadow on the ground. At a certain instant the ball is 5 m above the ground and falling at 4 m/s toward a spot on the ground 6 m from the base of the lamp post. How fast is the shadow moving along the ground at that instant? State units with your answer.



$$\frac{dy}{dt} = -4\ \frac{m}{s}$$

Find $\frac{dx}{dt}$ when $y = 5$ m.

By similar triangles: $\frac{x-6}{y} = \frac{x}{10}$

$$10x - 60 = xy$$

$$10x - xy = 60$$

$$x(10-y) = 60$$

$$x = \frac{60}{10-y}$$

$$\therefore \frac{dx}{dt} = \frac{60}{(10-y)^2}\left(-\frac{dy}{dt}\right)$$

When $y=5$: $\frac{dx}{dt} = \frac{-60}{(10-5)^2}\cdot(4) = \frac{-\cancel{60}^{12}}{\cancel{25}_5}\cdot 4 = \frac{-48}{5}\ \frac{m}{s}$.

$\therefore$ Shadow is moving toward lamp post at $\frac{48}{5}\ \frac{m}{s}$.