

Question 1:

(a)[3 points] Determine y' if $y = 5x^4 - \pi x + 6\sqrt{x} - \sqrt{2} = 5x^4 - \pi x + 6x^{\frac{1}{2}} - \sqrt{2}$

$$y' = 20x^3 - \pi + 3x^{-\frac{1}{2}}$$

(b)[3 points] Determine $\frac{dy}{dx}$ where $y = \left(3x^2 - \frac{1}{x}\right) \sec x = (3x^2 - x^{-1}) \sec x$

$$\frac{dy}{dx} = (6x + x^{-2}) \sec x + (3x^2 - x^{-1}) \sec x \tan x$$

(c)[4 points] Determine $f'(x)$ if $f(x) = \frac{\csc x - \sqrt[3]{x}}{\tan x + 2x^{1/2}} = \frac{\csc x - x^{\frac{1}{3}}}{\tan x + 2x^{\frac{1}{2}}}$

$$f'(x) = \frac{(\tan x + 2x^{\frac{1}{2}})(-\csc x \cot x - \frac{1}{3}x^{-\frac{2}{3}}) - (\csc x - x^{\frac{1}{3}})(\sec^2 x + x^{-\frac{1}{2}})}{(\tan x + 2x^{\frac{1}{2}})^2}$$

Question 2:

- (a) [5 points] A particle moving along a straight line has position at time t given by $s(t) = 2t^3 - 12t^2 + t + 2$. Here $t \geq 0$ is in seconds and position is in metres. Determine the velocity of the particle when the acceleration is 0 m/s^2 .

$$\text{velocity } v(t) = s'(t) = 6t^2 - 24t + 1$$

$$\text{acceleration } a(t) = s''(t) = 12t - 24$$

$$a(t) = 0 \Rightarrow 12t - 24 = 0$$

$$\Rightarrow t = \frac{24}{12} = 2 \text{ s.}$$

$$\therefore v(2) = 6(2)^2 - 24(2) + 1$$

$$= \boxed{-23 \frac{\text{m}}{\text{s}}}$$

- (b) [5 points] Determine the values of a and b for which the line $y = -2x + b$ is tangent to the parabola $y = ax^2$ when $x = 2$.

Since the tangent line $y = -2x + b$ and curve $y = ax^2$ share the point of contact,

$$-2(2) + b = a(2)^2$$

$$\therefore 4a - b = -4 \dots (*)$$

Also, $\left. \frac{d}{dx} [ax^2] \right|_{x=2} = -2$, the slope of the tangent line.

$$\therefore 2ax \Big|_{x=2} = -2$$

$$\therefore 4a = -2 \Rightarrow a = \frac{-2}{4} = -\frac{1}{2}$$

$$\text{From } (*): 4\left(-\frac{1}{2}\right) - b = -4 \Rightarrow b = -2 + 4 = 2$$

$$\therefore a = -\frac{1}{2}, b = 2$$

Question 3:

(a)[3 points] Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x + \sqrt{x}} = (x + x^{1/2})^{1/3}$

$$\frac{dy}{dx} = \frac{1}{3} (x + x^{1/2})^{-2/3} (1 + \frac{1}{2} x^{-1/2})$$

(b)[3 points] Find $\frac{dy}{dx}$ if $y = \sin(\sqrt{7x + \cos x})$

$$\frac{dy}{dx} = \cos(\sqrt{7x + \cos x}) \cdot \frac{1}{2} (7x + \cos x)^{-1/2} (7 - \sin x)$$

(c)[4 points] Compute $g''(\pi)$ if $g(\theta) = \theta^2 \cos \theta$

$$g'(\theta) = 2\theta \cos \theta - \theta^2 \sin \theta$$

$$g''(\theta) = 2 \cos \theta - 2\theta \sin \theta - 2\theta \sin \theta - \theta^2 \cos \theta$$

$$g''(\pi) = 2 \cos(\pi) - \cancel{2\pi \sin(\pi)} - \cancel{2\pi \sin(\pi)} - \pi^2 \cos(\pi)$$

$$= -2 + \pi^2$$

Question 4:

(a) [4 points] Determine $f'(x)$ if $f(x) = (1 - x^2) \cos\left(\frac{1}{x}\right) = (1 - x^2) \cos(x^{-1})$

$$f'(x) = -2x \cos(x^{-1}) + (1 - x^2) \sin(x^{-1}) x^{-2}$$

(b) [6 points] Find an equation of the tangent line to the curve $x \sin(xy - y) = \frac{x - y}{y}$ at the point (1, 1).

$$\frac{d}{dx} [x \sin(xy - y)] = \frac{d}{dx} \left[\frac{x - y}{y} \right]$$

$$1 \cdot \sin(xy - y) + x \cos(xy - y) \cdot (1 \cdot y + xy' - y') = \frac{y(1 - y') - (x - y)y'}{y^2}$$

At (1, 1):

$$\cancel{\sin(0)} + \cancel{\cos(0)} (1 + y' - y') = \frac{1 - y' - 0}{1}$$

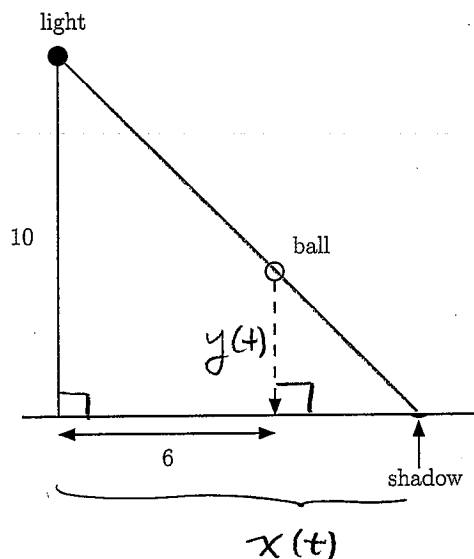
$$1 = 1 - y'$$

$$\therefore y' = 0$$

$$\therefore y - 1 = 0 \cdot (x - 1)$$

$$\boxed{\therefore y = 1}$$

Question 5 [10 points]: A ball is falling vertically to the ground near a 10 m tall lamp post. It is night time, so the light atop the lamp post casts the ball's shadow on the ground. At a certain instant the ball is 4 m above the ground and falling at 5 m/s toward a spot on the ground 6 m from the base of the lamp post. How fast is the shadow moving along the ground at that instant? State units with your answer.



$$\frac{dy}{dt} = -5 \frac{m}{s}$$

Find $\frac{dx}{dt}$ when $y=4$,

By similar triangles

$$\frac{x-6}{y} = \frac{x}{10}$$

$$\therefore 10x - 60 = xy$$

$$10x - xy = 60$$

$$x(10-y) = 60$$

$$x = \frac{60}{10-y}$$

$$\therefore \frac{dx}{dt} = \frac{-60}{(10-y)^2} \cdot \left(-\frac{dy}{dt}\right)$$

$$\text{When } y=4, \quad \frac{dx}{dt} = \frac{-60}{(10-4)^2} \cdot (-5) = \frac{-60}{36} \cdot 5 = -\frac{25}{3} \frac{m}{s}$$

\therefore Shadow is moving toward lamp post at $\frac{25}{3} \frac{m}{s}$.