Question 1:

(a) [5 points] Evaluate: $\lim_{x\to 25} \frac{5-\sqrt{x}}{x-25}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

$$\lim_{x\to 25} \frac{5-\sqrt{x}}{x-25} = \lim_{x\to 25} \frac{5-\sqrt{x}}{x-25} \cdot \frac{5+\sqrt{x}}{5+\sqrt{x}}$$

$$= \lim_{x\to 25} \frac{25-x}{(x-25)(5+\sqrt{x})}$$

$$= \lim_{x\to 25} \frac{-\cancel{(x-25)}}{\cancel{(x-25)}(5+\sqrt{x})}$$

$$= \frac{-1}{10}$$

(b) [5 points] Evaluate: $\lim_{x\to -4^-} \frac{4x+16}{|x+4|}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

Since $x \to -4^-$, $x+4 < 0$, so $|x+4| = -(x+4)$

$$\therefore \lim_{x\to -4^-} \frac{4x+16}{|x+4|} = \lim_{x\to -4^-} \frac{4\cancel{(x+4)}}{-\cancel{(x+4)}}$$

$$= -4$$

Question 2:

(a) [5 points] Evaluate: $\lim_{x\to 3} \frac{\sqrt{x^2+16}-3}{x^2-3x-3}$

$$\lim_{x\to 3} \frac{\sqrt{x^2+16}-3}{x^2-3x-3} = \frac{\sqrt{9+16}-3}{\cancel{9}-\cancel{9}-3}$$

$$= \frac{5-3}{-3}$$

$$= \frac{-2}{3}$$

(b) [5 points] Evaluate: $\lim_{x\to -4} \frac{x^2+5x+4}{x^2-x-20}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

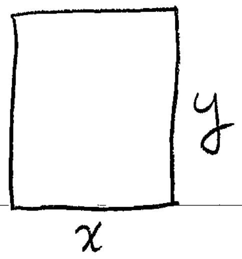
$$\lim_{x\to -4} \frac{x^2+5x+4}{x^2-x-20} = \lim_{x\to -4} \frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x-5)}$$

$$= \frac{-3}{-9}$$

$$= \frac{1}{3}.$$

Question 3:

(a) [**3 points**] A rectangle has area 16 m^2. Express the perimeter P as a function of the length x of one of its sides.



$$xy = 16 \Rightarrow y = \frac{16}{x}$$

$$P = 2x + 2y = 2x + 2\left(\frac{16}{x}\right) = 2x + \frac{32}{x}$$

$$\therefore P = 2x + \frac{32}{x}.$$

(b) [**3 points**] Let $H(x) = \sec^2(\sqrt{x^2-1})$ and $h(x) = x^2$. Find functions f and g so that $H = f \circ g \circ h$. (There are several possible correct answers.)

$$g(x) = \sqrt{x-1}, \quad f(x) = \sec^2 x$$

$\underline{\underline{\text{or}}}$ $$g(x) = \sec(\sqrt{x-1}), \quad f(x) = x^2$$

(c) [**4 points**] Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{x}{\sqrt{x+2}}$. Determine, simplify, and find the domain of $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x)) = \frac{1}{\underbrace{\left(\frac{x}{\sqrt{x+2}}\right)^2}_{*}} = \frac{x+2}{x^2}$$

Using $*$, must have $x \neq 0$, $x+2 > 0$

$\therefore$ $x \neq 0$, $x > -2$.

$\therefore$ Domain of $f \circ g$ is $(-2, 0) \cup (0, \infty)$.

Question 4:

(a) [**5 points**] Evaluate: $\lim_{x\to\pi} \frac{\sin^2\left(\frac{x}{2}\right)}{x-\pi}$

As $x \to \pi$, $\sin^2(\frac{x}{2}) \to 1$ while $x-\pi \to 0$.

$\therefore \lim_{x\to\pi} \frac{\sin^2(\frac{x}{2})}{x-\pi}$ does not exist.

(b) [**5 points**] Let $f(x) = \frac{1}{x-3}$. Evaluate and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$.

$$\frac{f(a+h)-f(a)}{h} = \frac{1}{h}\left[\frac{1}{a+h-3} - \frac{1}{a-3}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{a}-\cancel{3}-\cancel{a}-h+\cancel{3}}{(a+h-3)(a-3)}\right]$$

$$= \frac{-\cancel{h}}{\cancel{h}(a+h-3)(a-3)}$$

$$= \frac{-1}{(a+h-3)(a-3)}.$$

Question 5:

(a) [5 points] Evaluate: $\lim_{t\to 0} \frac{\sin(5t)}{\tan(7t)}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

$$\lim_{t\to 0} \frac{\sin(5t)}{\tan(7t)} = \lim_{t\to 0} \frac{\sin(5t)}{\left(\frac{\sin(7t)}{\cos(7t)}\right)}$$

$$= \lim_{t\to 0} \underbrace{\frac{\sin(5t)}{5t}}_{\to 1} \cdot \underbrace{\frac{\cos(7t)}{\left(\frac{\sin(7t)}{7t}\right)}}_{\to 1} \cdot \frac{5\cancel{t}}{7\cancel{t}}$$

$$= 1 \cdot \frac{1}{1} \cdot \frac{5}{7}$$

$$= \frac{5}{7}$$

(b) [5 points] Evaluate: $\lim_{x\to 0} x^4 \sin\left(\frac{4}{x}\right)$
(the Squeeze Theorem may help here).

$$-x^4 \le x^4 \sin\left(\frac{4}{x}\right) \le x^4$$

Since $\lim_{x\to 0}(-x^4) = 0 = \lim_{x\to 0} x^4$,

by the Squeeze Theorem,

$$\lim_{x\to 0} x^4 \sin\left(\frac{4}{x}\right) = 0.$$