

(1) [6 points] Determine the linearization (or linear approximation) $L(x)$ of $f(x) = \sin x$ at $a = \pi/6$.

$$f(x) = \sin(x) \quad ; \quad f(a) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$
$$f'(x) = \cos(x) \quad ; \quad f'(a) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{6}\right)$$

(2) [9 points] Use a linear approximation to estimate $\sqrt{9.1}$.

Here $f(x) = x^{1/2}$, $a = 9$.

$$f(a) = 9^{1/2} = 3$$

$$f'(x) = \frac{1}{2} x^{-1/2} \quad ; \quad f'(a) = \frac{1}{2} (9)^{-1/2} = \frac{1}{6}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$
$$= 3 + \frac{1}{6}(x-9)$$

$$\therefore \sqrt{9.1} \approx L(9.1) = 3 + \frac{1}{6}(9.1-9)$$
$$= 3 + \frac{1}{6}(0.1)$$
$$= 3 + \frac{1}{60}$$
$$= \frac{181}{60}$$