

(1) [7 points] Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+2}{x+3}$.

(i) Determine and simplify $(f \circ g)(x)$.

(ii) Determine the domain of $(f \circ g)(x)$.

$$(i) (f \circ g)(x) = f(g(x))$$

$$= g(x) + \frac{1}{g(x)}$$

$$= \frac{x+2}{x+3} + \frac{1}{\left(\frac{x+2}{x+3}\right)} \quad \left. \vphantom{\frac{x+2}{x+3}} \right\} *$$

$$= \frac{x+2}{x+3} + \frac{x+3}{x+2}$$

$$= \frac{(x+2)^2 + (x+3)^2}{(x+3)(x+2)}$$

$$= \frac{x^2 + 4x + 4 + x^2 + 6x + 9}{(x+3)(x+2)}$$

$$= \frac{2x^2 + 10x + 13}{(x+3)(x+2)}$$

(ii) Using (*), domain of $f \circ g$ is all real x excluding $x = -3$, $x = -2$, i.e. $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$.

(2) [8 points] Neatly sketch the graph of a function $f(x)$ which satisfies the following properties (there are many possible correct answers):

$$\lim_{x \rightarrow 1} f(x) = 3, \quad \lim_{x \rightarrow 4^-} f(x) = 3, \quad \lim_{x \rightarrow 4^+} f(x) = -3, \quad f(1) = 1, \quad f(4) = -1$$

