

Math 121 - Basic Derivative Formulas

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Derivative Rules

Assumptions

In the following, suppose:

- c represents a constant (a fixed number)
- The functions $f(x)$ and $g(x)$ are both differentiable. That is,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{and} \quad g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

both exist

Constant Rule

- $\frac{d}{dx} [c] = 0$

- In words: *The derivative of a constant is zero*

- Example: $\frac{d}{dx} [\sqrt{2\pi}] = 0$

- Proof: let $f(x) = c$. Then

$$\frac{d}{dx} [c] = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{c - c}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

Power Rule

- If n is any real number, $\frac{d}{dx} [x^n] = nx^{n-1}$

- Example: $\frac{d}{dx} [x^{11}] = 11x^{10}$

- Proof (in the case where n is a positive integer): let $f(x) = x^n$.

$$\begin{aligned}\frac{d}{dx} [x^n] &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^n + nx^{n-1}h + (\text{terms with factor of } h^2) + \dots - x^n}{h} \\ &= \lim_{h \rightarrow 0} nx^{n-1} + (\text{terms with factor of } h) \\ &= nx^{n-1}\end{aligned}$$

Power Rule, case $n = 1$

- $\frac{d}{dx} [x] = 1$

- Why? $\frac{d}{dx} [x^1] = 1 \cdot x^0 = 1$

Constant Multiple Rule

- $\frac{d}{dx} [c \cdot f(x)] = c \cdot \frac{d}{dx} [f(x)]$
- In words: *The derivative of a constant times a function is the constant times the derivative of the function*

Constant Multiple Rule

- Proof of Constant Multiple Rule:

$$\begin{aligned}\frac{d}{dx} [c \cdot f(x)] &= \lim_{h \rightarrow 0} \frac{c \cdot f(x+h) - c \cdot f(x)}{h} \\ &= \lim_{h \rightarrow 0} c \cdot \frac{f(x+h) - f(x)}{h} \\ &= c \cdot \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= c \cdot \frac{d}{dx} [f(x)]\end{aligned}$$

- Example: $\frac{d}{dx} [3x^{11}] = 3 \cdot \frac{d}{dx} [x^{11}] = 3(11x^{10}) = 33x^{10}$

Sum Rule

- $\frac{d}{dx} [f(x) + g(x)] = \frac{d}{dx} [f(x)] + \frac{d}{dx} [g(x)]$

- In words: *The derivative of a sum is the sum of the derivatives*

Sum Rule

Proof of the Sum Rule:

$$\begin{aligned} & \frac{d}{dx} [f(x) + g(x)] \\ = & \lim_{h \rightarrow 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\ = & \lim_{h \rightarrow 0} \frac{[f(x+h) - f(x)] + [g(x+h) - g(x)]}{h} \\ = & \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right) \\ = & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\ = & \frac{d}{dx} [f(x)] + \frac{d}{dx} [g(x)] \end{aligned}$$

Difference Rule

- $\frac{d}{dx} [f(x) - g(x)] = \frac{d}{dx} [f(x)] - \frac{d}{dx} [g(x)]$
- In words: *The derivative of a difference is the difference of the derivatives*
- Example:

$$\begin{aligned}\frac{d}{dx} \left[x^3 - 2x^{1/2} + \frac{x}{2} \right] &= \frac{d}{dx} [x^3] - \frac{d}{dx} [2x^{1/2}] + \frac{d}{dx} \left[\frac{1}{2}x \right] \\ &= 3x^2 - 2 \frac{d}{dx} [x^{1/2}] + \frac{1}{2} \frac{d}{dx} [x] \\ &= 3x^2 - 2 \left(\frac{1}{2} x^{-1/2} \right) + \frac{1}{2} (1) \\ &= 3x^2 - x^{-1/2} + \frac{1}{2}\end{aligned}$$

Sine and Cosine Rule

- $\frac{d}{dx} [\sin(x)] = \cos(x)$
- $\frac{d}{dx} [\cos(x)] = -\sin(x)$
- Example:

$$\begin{aligned}\frac{d}{dx} \left[4 \cos(x) + \frac{\sin(x)}{\pi} \right] &= 4 \frac{d}{dx} [\cos(x)] + \frac{1}{\pi} \frac{d}{dx} [\sin(x)] \\ &= -4 \sin(x) + \frac{1}{\pi} \cos(x)\end{aligned}$$