Question 1:

(a)[3 points] Determine y' if $y = 5x^4 - \pi x + 6\sqrt{x} - \sqrt{2}$ $= 5x^4 - \pi x + 6x^{1/2} - \sqrt{2}$

$$y' = 20x^3 - \pi + 3x^{-\frac{1}{2}}$$

(b)[3 points] Determine $\frac{dy}{dx}$ where $y = \left(3x^2 + \frac{1}{x}\right)\csc x$ $= (3x^2 + x^{-1})\csc x$

$$\therefore \frac{dy}{dx} = (6x - x^{-2})\csc x + (3x^2 + x^{-1})(-\csc x \cot x)$$

(c)[4 points] Determine $f'(x)$ if $f(x) = \frac{\sec x - \sqrt[3]{x}}{\tan x - 2x^{1/2}}$ $= \frac{\sec x - x^{\frac{1}{3}}}{\tan x - 2x^{1/2}}$

$$f'(x) = \frac{(\tan x - 2x^{1/2})(\sec x \tan x - \frac{1}{3}x^{-2/3}) - (\sec x - x^{1/3})(\sec^2 x - x^{-1/2})}{(\tan x - 2x^{1/2})^2}$$

Question 2:

(a) [3 points] Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x + \sqrt{x}}$ $= \left(x + x^{\frac{1}{2}}\right)^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3}\left(x + x^{\frac{1}{2}}\right)^{-\frac{2}{3}} \left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)$$

(b) [3 points] Find $\frac{dy}{dx}$ if $y = \sin(\sqrt{7x + \cos x})$ $= \sin\left(\left(7x + \cos x\right)^{\frac{1}{2}}\right)$

$$\frac{dy}{dx} = \cos\left(\left(7x + \cos x\right)^{\frac{1}{2}}\right) \cdot \frac{1}{2}\left(7x + \cos x\right)^{-\frac{1}{2}} \left(7 - \sin x\right)$$

(c) [4 points] Compute $g''(\pi)$ if $g(\theta) = \theta^2 \cos\theta$

$$g'(\theta) = 2\theta\cos\theta - \theta^2 \sin\theta$$

$$g''(\theta) = 2\cos\theta - 2\theta\sin\theta - 2\theta\sin\theta - \theta^2\cos\theta$$

$$g''(\pi) = 2\cos\pi - \cancelto{0}{2\pi\sin\pi} - \cancelto{0}{2\pi\sin\pi} - \pi^2\cos\pi$$

$$= -2 + \pi^2$$

Question 3:

(a) [5 points] Determine the values of a and b for which the line $y = -2x + b$ is tangent to the parabola $y = ax^2$ when $x = 2$.

Since the tangent line $y = -2x+b$ and curve $y = ax^2$ share the point of contact,

$$-2(2)+b = a(2)^2$$

$$\therefore\ 4a - b = -4 \quad \ldots (*)$$

Also, $\left.\frac{d}{dx}[ax^2]\right|_{x=2} = -2$, the slope of the tangent line

$$\therefore\ \left.2ax\right|_{x=2} = -2$$

$$\therefore\ 4a = -2 \Rightarrow a = \frac{-2}{4} = \frac{-1}{2}$$

From $(*)$: $4(\frac{-1}{2}) - b = -4 \Rightarrow b = -2+4 = 2$

$$\boxed{\therefore\ a = \frac{-1}{2},\ b = 2}$$

(b) [5 points] A particle moving along a straight line has position at time t given by $s(t) = 2t^3 - 12t^2 + t + 2$. Here $t \geq 0$ is in seconds and position is in metres. Determine the velocity of the particle when the acceleration is 0 m/s^2.

velocity $v(t) = s'(t) = 6t^2 - 24t + 1$

acceleration $a(t) = s''(t) = 12t - 24$

$$a(t) = 0 \Rightarrow 12t - 24 = 0$$

$$\Rightarrow t = \frac{24}{12} = 2 \text{ s,}$$

$$\therefore\ v(2) = 6(2)^2 - 24(2) + 1$$

$$= \boxed{-23\ \frac{m}{s}}$$

Question 4:

(a) [4 points] Determine $f'(x)$ if $f(x) = (1-x^2)\sin\left(\frac{1}{x}\right)$ $= (1-x^2)\sin(x^{-1})$

$$f'(x) = -2x\sin(x^{-1}) + (1-x^2)\cos(x^{-1})(-x^{-2})$$

(b) [6 points] Find an equation of the tangent line to the curve $x\sin(y-xy) = \frac{x-y}{y}$ at the point $(1,1)$.

$$\frac{d}{dx}\left[x\sin(y-xy)\right] = \frac{d}{dx}\left[\frac{x-y}{y}\right]$$

$$1\cdot\sin(y-xy) + x\cos(y-xy)(y'-y-xy') = \frac{(y)(1-y')-(x-y)y'}{y^2}$$

at $(1,1)$:

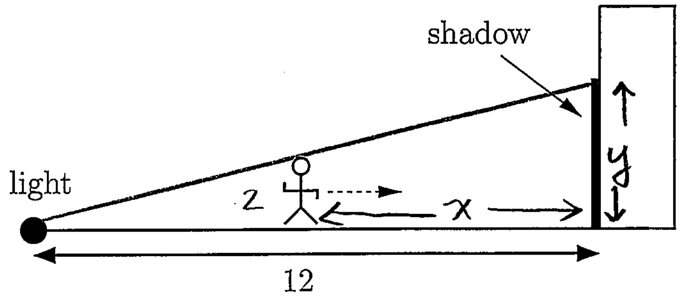
$$1\cdot\cancel{\sin(0)}^{0} + 1\cdot\cos(0)(\cancel{y'}-1-\cancel{y'}) = \frac{(1)(1-y')-\cancel{(1-1)y'}^{0}}{(1)^2}$$

$$-1 = 1-y'$$

$$\therefore\ y' = 2$$

$$\boxed{\therefore\ y-1 = 2(x-1)}$$

Question 5 [10 points]: A splotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight to the building at a speed of 1/2 m/s, how fast is the length of his shadow on the building decreasing when he is 6 m from the building?



$$\frac{dx}{dt} = \frac{-1}{2}\ \frac{m}{s}.$$

Find $\frac{dy}{dt}$ when $x = 6$ m.

Using similar triangles, $\frac{2}{12-x} = \frac{y}{12}$

$$\therefore\ y = \frac{24}{12-x}$$

$$\therefore\ \frac{dy}{dt} = \frac{-24}{(12-x)^2} \cdot \left(-\frac{dx}{dt}\right)$$

When $x = 6$: $\frac{dy}{dt} = \frac{-24}{(12-6)^2} \cdot \left(\frac{1}{2}\right)$

$$= \frac{-1}{3}\ \frac{m}{s}$$

$\therefore$ Shadow is decreasing by $\frac{1}{3}\ \frac{m}{s}$.