**Question 1:**

**(a)[3 points]** Determine $y'$ if $y = 5x^4 - \pi x + 6\sqrt{x} - \sqrt{2}$ $= 5x^4 - \pi x + 6x^{\frac{1}{2}} - \sqrt{2}$

$$y' = 20x^3 - \pi + 3x^{-\frac{1}{2}}$$

**(b)[3 points]** Determine $\frac{dy}{dx}$ where $y = \left(3x^2 - \frac{1}{x}\right)\sec x$ $= (3x^2 - x^{-1})\sec x$

$$\frac{dy}{dx} = (6x + x^{-2})\sec x + (3x^2 - x^{-1})\sec x \tan x$$

**(c)[4 points]** Determine $f'(x)$ if $f(x) = \dfrac{\csc x - \sqrt[3]{x}}{\tan x + 2x^{1/2}}$ $= \dfrac{\csc x - x^{\frac{1}{3}}}{\tan x + 2x^{1/2}}$

$$f'(x) = \frac{(\tan x + 2x^{1/2})(-\csc x \cot x - \frac{1}{3}x^{-2/3}) - (\csc x - x^{1/3})(\sec^2 x + x^{-1/2})}{(\tan x + 2x^{1/2})^2}$$

Question 2:

**(a)[5 points]** A particle moving along a straight line has position at time $t$ given by $s(t) = 2t^3 - 12t^2 + t + 2$. Here $t \geq 0$ is in seconds and position is in metres. Determine the velocity of the particle when the acceleration is 0 m/s$^2$.

velocity $v(t) = s'(t) = 6t^2 - 24t + 1$

acceleration $a(t) = s''(t) = 12t - 24$

$a(t) = 0 \Rightarrow 12t - 24 = 0$

$\Rightarrow t = \frac{24}{12} = 2$ s.

$\therefore v(2) = 6(2)^2 - 24(2) + 1$

$= \boxed{-23 \frac{m}{s}}$

**(b)[5 points]** Determine the values of $a$ and $b$ for which the line $y = -2x + b$ is tangent to the parabola $y = ax^2$ when $x = 2$.

Since the tangent line $y = -2x + b$ and curve $y = ax^2$ share the point of contact,

$-2(2) + b = a(2)^2$

$\therefore 4a - b = -4 \quad \ldots (*)$

Also, $\frac{d}{dx}\left[ax^2\right]\Big|_{x=2} = -2$, the slope of the tangent line.

$\therefore 2ax\Big|_{x=2} = -2$

$\therefore 4a = -2 \Rightarrow a = \frac{-2}{4} = -\frac{1}{2}$

From $(*)$: $4(\frac{-1}{2}) - b = -4 \Rightarrow b = -2 + 4 = 2$

$\boxed{\therefore a = -\frac{1}{2}, \ b = 2}$

**Question 3:**

**(a)[3 points]** Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x+\sqrt{x}}$ $= \left(x + x^{\frac{1}{2}}\right)^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3}\left(x + x^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)$$

**(b)[3 points]** Find $\frac{dy}{dx}$ if $y = \sin(\sqrt{7x + \cos x})$

$$\frac{dy}{dx} = \cos\left(\sqrt{7x+\cos x}\right) \cdot \frac{1}{2}\left(7x + \cos x\right)^{-\frac{1}{2}}\left(7 - \sin x\right)$$

**(c)[4 points]** Compute $g''(\pi)$ if $g(\theta) = \theta^2 \cos\theta$

$$g'(\theta) = 2\theta\cos\theta - \theta^2\sin\theta$$

$$g''(\theta) = 2\cos\theta - 2\theta\sin\theta - 2\theta\sin\theta - \theta^2\cos\theta$$

$$g''(\pi) = 2\cos(\pi) - 2\pi\cancelto{0}{\sin(\pi)} - 2\pi\cancelto{0}{\sin(\pi)} - \pi^2\cos(\pi)$$

$$= -2 + \pi^2$$

Question 4:

**(a)[4 points]** Determine $f'(x)$ if $f(x) = (1-x^2)\cos\left(\frac{1}{x}\right)$ $= (1-x^2)\cos(x^{-1})$

$$f'(x) = -2x\cos(x^{-1}) + (1-x^2)\sin(x^{-1})\,x^{-2}$$

**(b)[6 points]** Find an equation of the tangent line to the curve $x\sin(xy-y) = \frac{x-y}{y}$ at the point $(1,1)$.

$$\frac{d}{dx}\left[x\sin(xy-y)\right] = \frac{d}{dx}\left[\frac{x-y}{y}\right]$$

$$1\cdot\sin(xy-y) + x\cos(xy-y)\cdot(1\cdot y + xy' - y') = \frac{y(1-y') - (x-y)y'}{y^2}$$

At $(1,1)$:

$$\cancel{\sin}(0)^{\,0} + \cancel{\cos}(0)^{\,1}(1+\cancel{y'}-\cancel{y'}) = \frac{1-y'-0}{1}$$

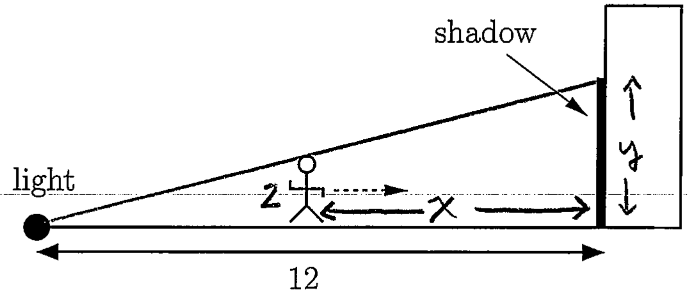
$$1 = 1-y'$$

$$\therefore\ y' = 0$$

$$\therefore\ y-1 = 0\cdot(x-1)$$

$$\boxed{\therefore\ y=1}$$

**Question 5 [10 points]:** A splotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight to the building at a speed of 2 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?




$\frac{dx}{dt} = -2 \frac{m}{s}$.

Find $\frac{dy}{dt}$ when $x = 4$ m.

Using similar triangles, $\frac{2}{12-x} = \frac{y}{12}$

$\therefore y = \frac{24}{12-x}$

$\therefore \frac{dy}{dt} = \frac{-24}{(12-x)^2} \cdot \left(-\frac{dx}{dt}\right)$

At $x = 4$ : $\frac{dy}{dt} = \frac{-24}{(12-4)^2} \cdot (2)$

$= -\frac{3}{4} \frac{m}{s}$

$\therefore$ Shadow is decreasing by $\frac{3}{4} \frac{m}{s}$.