Question 1:

(a) [5 points] Evaluate: $\lim_{t\to 0} \frac{\sin(5t)}{\tan(3t)}$ $\frac{\to 0}{\to 0}$

$$\lim_{t\to 0} \frac{\sin(5t)}{\tan(3t)} = \lim_{t\to 0} \frac{\sin(5t)}{\left(\frac{\sin(3t)}{\cos(3t)}\right)}$$

$$= \lim_{t\to 0} \underbrace{\frac{\sin(5t)}{5t}}_{\to 1} \cdot \frac{\cos(3t)}{\underbrace{\left(\frac{\sin(3t)}{3t}\right)}_{\to 1}} \cdot \frac{5\cancel{t}}{3\cancel{t}}$$

$$= 1 \cdot \frac{1}{1} \cdot \frac{5}{3}$$

$$= \frac{5}{3}.$$

(b) [5 points] Evaluate: $\lim_{x\to 0} x^8 \cos\left(\frac{8}{x}\right)$
(the Squeeze Theorem may help here).

$$-x^8 \le x^8 \cos\left(\frac{8}{x}\right) \le x^8$$

Since $\lim_{x\to 0} (-x^8) = 0 = \lim_{x\to 0} x^8$,

by the Squeeze Theorem, $\lim_{x\to 0} x^8 \cos\left(\frac{8}{x}\right) = 0$.

Question 2:

(a) **[3 points]** Recall that a circle of radius r has area $A = \pi r^2$ and circumference $C = 2\pi r$. Express the circumference C as a function of the area A.

$A = \pi r^2$, so $r = \left(\frac{A}{\pi}\right)^{1/2}$

$\therefore\ C = 2\pi r = 2\pi\left(\frac{A}{\pi}\right)^{1/2} = 2\pi^{1/2}A^{1/2} = 2\sqrt{\pi A}$

$\therefore\ C = 2\sqrt{\pi A}$

(b) **[3 points]** Let $H(x) = \cot^2(\sqrt{x^2+2})$ and $h(x) = x^2$. Find functions f and g so that $H = f \circ g \circ h$. (There are several possible correct answers.)

$g(x) = x+2$, $f(x) = \cot^2(\sqrt{x})$

or $g(x) = \sqrt{x+2}$, $f(x) = \cot^2 x$

(c) **[4 points]** Let $f(x) = \dfrac{2}{x^2}$ and $g(x) = \dfrac{x}{\sqrt{3-x}}$. Determine, simplify, and find the domain of $(f \circ g)(x)$.

$(f\circ g)(x) = f(g(x)) = \dfrac{2}{\underbrace{\left(\frac{x}{\sqrt{3-x}}\right)^2}_{*}} = \dfrac{2(3-x)}{x^2} = \dfrac{6-2x}{x^2}$

Using $*$, we must have $x \neq 0$, $3-x>0$

$\therefore\ x \neq 0$, $3 > x$

$\therefore$ Domain of $f\circ g$ is $(-\infty,0)\cup(0,3)$

Question 3:

(a) [5 points] Evaluate: $\lim_{x \to \pi} \frac{\sin^2\left(\frac{x}{2}\right)}{x - \pi}$

As $x \to \pi$, $\sin^2(\frac{x}{2}) \to 1$ while $x - \pi \to 0$.

$\therefore \lim_{x \to \pi} \frac{\sin^2(\frac{x}{2})}{x - \pi}$ does not exist.

(b) [5 points] Let $f(x) = \frac{1}{x+3}$. Evaluate and simplify the difference quotient $\frac{f(a+h) - f(a)}{h}$.

$$\frac{f(a+h) - f(a)}{h} = \frac{1}{h}\left[\frac{1}{a+h+3} - \frac{1}{a+3}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{a} \cancel{+3} - \cancel{a} - h \cancel{-3}}{(a+h+3)(a+3)}\right]$$

$$= \frac{-h}{h(a+h+3)(a+3)}$$

$$= \frac{-1}{(a+h+3)(a+3)}.$$

Question 4:

(a) [5 points] Evaluate: $\lim_{x\to 36} \frac{6-\sqrt{x}}{x-36}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

$$\lim_{x\to 36} \frac{6-\sqrt{x}}{x-36} = \lim_{x\to 36} \frac{6-\sqrt{x}}{x-36} \cdot \frac{6+\sqrt{x}}{6+\sqrt{x}}$$

$$= \lim_{x\to 36} \frac{36-x}{(x-36)(6+\sqrt{x})}$$

$$= \lim_{x\to 36} \frac{-\cancel{(x-36)}}{\cancel{(x-36)}(6+\sqrt{x})}$$

$$= \frac{-1}{6+6}$$

$$= \frac{-1}{12}$$

(b) [5 points] Evaluate: $\lim_{x\to -5^-} \frac{5x+25}{|x+5|}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

Since $x \to -5^-$, $x+5 < 0$, so $|x+5| = -(x+5)$

$$\therefore \lim_{x\to -5^-} \frac{5x+25}{|x+5|} = \lim_{x\to -5^-} \frac{5\cancel{(x+5)}}{-\cancel{(x+5)}}$$

$$= -5$$

Question 5:

(a)[5 points] Evaluate: $\lim_{x \to 3} \frac{x^2 - 3x - 3}{\sqrt{x^2+16} - 3}$

$$\lim_{x \to 3} \frac{x^2-3x-3}{\sqrt{x^2+16}-3} = \frac{9-9-3}{\sqrt{9+16}-3}$$

$$= \frac{-3}{5-3}$$

$$= \frac{-3}{2}$$

(b)[5 points] Evaluate: $\lim_{x \to -6} \frac{x^2+8x+12}{x^2+5x-6}$ $\begin{matrix} \to 0 \\ \to 0 \end{matrix}$

$$\lim_{x \to -6} \frac{x^2+8x+12}{x^2+5x-6} = \lim_{x \to -6} \frac{\cancel{(x+6)}(x+2)}{\cancel{(x+6)}(x-1)}$$

$$= \frac{-4}{-7}$$

$$= \frac{4}{7}.$$