Question 1:

(a)[5 points] Evaluate: $\lim_{x\to\pi} \frac{\sin^2\left(\frac{x}{2}\right)}{x-\pi}$

As $x \to \pi$, $\sin^2\left(\frac{x}{2}\right) \to 1$ and $x - \pi \to 0$,

so $\lim_{x\to\pi} \frac{\sin^2\left(\frac{x}{2}\right)}{x-\pi}$ does not exist.

(b)[5 points] Let $f(x) = \frac{1}{x+2}$. Evaluate and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$.

$$\frac{f(a+h)-f(a)}{h} = \frac{1}{h}\left[\frac{1}{a+h+2} - \frac{1}{a+2}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{a}+\cancel{2}-\cancel{a}-h-\cancel{2}}{(a+h+2)(a+2)}\right]$$

$$= \frac{-h}{h(a+h+2)(a+2)}$$

$$= \frac{-1}{(a+h+2)(a+2)}$$

Question 2:

(a)[**5 points**] Evaluate: $\lim_{t\to 0} \frac{\sin(7t)}{\tan(3t)}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

$$\lim_{t\to 0} \frac{\sin(7t)}{\tan(3t)} = \lim_{t\to 0} \frac{\sin(7t)}{\left(\frac{\sin(3t)}{\cos(3t)}\right)}$$

$$= \lim_{t\to 0} \underbrace{\frac{\sin(7t)}{7t}}_{\to 1} \cdot \frac{\cos(3t)}{\underbrace{\left(\frac{\sin(3t)}{3t}\right)}_{\to 1}} \cdot \frac{7\cancel{t}}{3\cancel{t}}$$

$$= \frac{7}{3}$$

(b)[**5 points**] Evaluate: $\lim_{x\to 0} x^6 \cos\left(\frac{6}{x}\right)$
(the Squeeze Theorem may help here).

$$-x^6 \le x^6 \cos\left(\frac{6}{x}\right) \le x^6$$

since $\lim_{x\to 0} (-x^6) = 0 = \lim_{x\to 0} x^6$,

by the Squeeze Theorem, $\lim_{x\to 0} x^6 \cos\left(\frac{6}{x}\right) = 0.$

Question 3:

(a) [5 points] Evaluate: $\lim_{x \to 16} \frac{4-\sqrt{x}}{x-16}$ $\begin{matrix} \to 0 \\ \to 0 \end{matrix}$

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16} = \lim_{x\to 16} \frac{4-\sqrt{x}}{x-16} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}}$$

$$= \lim_{x\to 16} \frac{16-x}{(x-16)(4+\sqrt{x})}$$

$$= \lim_{x\to 16} \frac{-\cancel{(x-16)}}{\cancel{(x-16)}(4+\sqrt{x})}$$

$$= \frac{-1}{4+\sqrt{16}}$$

$$= \frac{-1}{8}$$

(b) [5 points] Evaluate: $\lim_{x \to -3^-} \frac{3x+9}{|x+3|}$ $\begin{matrix} \to \\ \to \end{matrix} \frac{0}{0}$

Since $x \to -3^-$, $x < -3$, so $x+3 < 0$, so $|x+3| = -(x+3)$.

$$\therefore \lim_{x\to -3^-} \frac{3x+9}{|x+3|} = \lim_{x\to -3^-} \frac{3\cancel{(x+3)}}{-\cancel{(x+3)}}$$

$$= -3$$

Question 4:

(a)[5 points] Evaluate: $\lim_{x\to 4} \frac{\sqrt{x^2+9}-4}{x^2-3x-3}$

$$\lim_{x\to 4} \frac{\sqrt{x^2+9}-4}{x^2-3x-3} = \frac{\sqrt{16+9}-4}{16-12-3} = \frac{1}{1} = 1$$

(b)[5 points] Evaluate: $\lim_{x\to -5} \frac{x^2+3x-10}{x^2+7x+10} \begin{matrix}\to 0 \\ \to 0\end{matrix}$

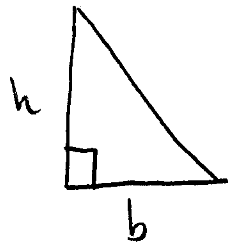
$$\lim_{x\to -5} \frac{x^2+3x-10}{x^2+7x+10} = \lim_{x\to -5} \frac{(x-2)\cancel{(x+5)}}{(x+2)\cancel{(x+5)}}$$

$$= \frac{-7}{-3}$$

$$= \frac{7}{3}.$$

Question 5:

(a) [**3 points**] A right angle triangle has base b and height h which add to 12. Express the area A of the triangle as a function of the base b.



$b + h = 12$

$\therefore\ h = 12 - b.$

$A = \frac{1}{2} bh = \frac{1}{2} b(12-b)$

$\therefore\ A = \frac{1}{2} b(12-b).$

(b) [**3 points**] Let $H(x) = \csc^2(\sqrt{x^2+1})$ and $h(x) = x^2$. Find functions f and g so that $H = f \circ g \circ h$. (There are several possible correct answers.)

$g(x) = \sqrt{x+1}\ ,\quad f(x) = \csc^2 x$

<u>or</u> $\quad g(x) = \csc(\sqrt{x+1})\ ,\quad f(x) = x^2$

(c) [**4 points**] Let $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{x}{\sqrt{x-1}}$. Determine, simplify, and find the domain of $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x)) = \frac{1}{\underbrace{\left(\frac{x}{\sqrt{x-1}}\right)^2}_{*}} = \frac{x-1}{x^2}$$

Using $*$, must have $x \neq 0$ and $x - 1 > 0$,

<u>i.e.</u> $x \neq 0$ and $x > 1$

$\therefore$ domain of $f \circ g$ is $(1, \infty)$.