Name:
Stu#: SOLUTIONS

(1) [6 points] Determine the linearization (or linear approximation) $L(x)$ of $f(x) = \sin x$ at $a = \pi/3$.

$f(x) = \sin(x)$; $f(a) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$f'(x) = \cos(x)$; $f'(a) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\therefore L(x) = f(a) + f'(a)(x-a)$

$$\boxed{L(x) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right).}$$

(2) [9 points] Use a linear approximation to estimate $\sqrt{15.9}$.

Here $f(x) = x^{1/2}$, $a = 16$.

$f(a) = 16^{1/2} = 4$

$f'(x) = \frac{1}{2}x^{-1/2}$; $f'(a) = \frac{1}{2}(16)^{-\frac{1}{2}} = \frac{1}{8}$

$\therefore L(x) = f(a) + f'(a)(x-a)$

$= 4 + \frac{1}{8}(x-16)$

$\therefore \sqrt{15.9} \approx L(15.9) = 4 + \frac{1}{8}(15.9-16)$

$= 4 + \frac{1}{8}\left(\frac{-1}{10}\right)$

$= \frac{320-1}{80}$

$= \boxed{\frac{319}{80}}$