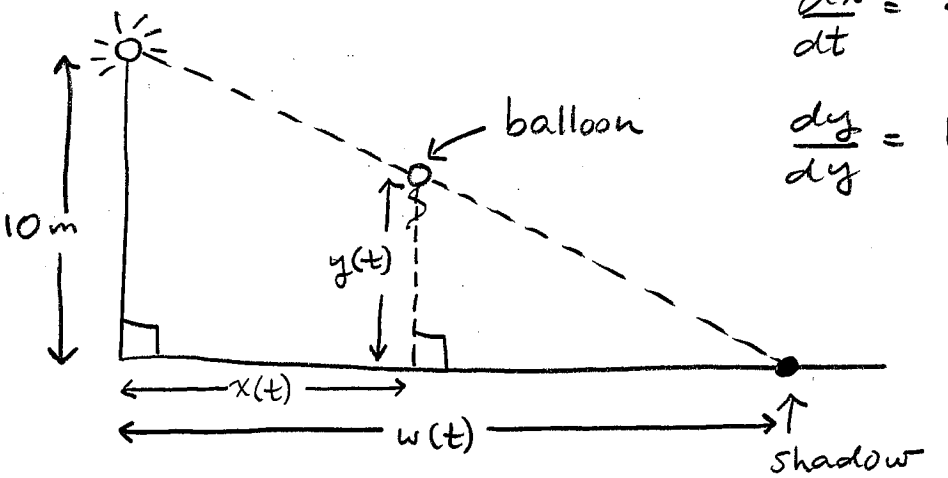
The Balloon Shadow Problem

A balloon is released from ground level at night near a 10 m tall lamp post. A few moments later the balloon is 6 m above the ground and rising vertically at 1 m/s, while the balloon's horizontal distance to the lamp post is 5 m and the wind is increasing that distance by 1/2 m/s. How fast is the balloon's shadow moving along the ground at that same instant?



$\frac{dx}{dt} = \frac{1}{2}\ \frac{m}{s}$

$\frac{dy}{dt} = 1\ \frac{m}{s}$.

Let $x(t)$ = horizontal distance to lamp post at time t
$y(t)$ = vertical distance to ground at time t
$w(t)$ = distance between shadow and lamp post at time t.

Find $\frac{dw}{dt}$ when $x = 5$ m & $y = 6$ m.

Using similar triangles:

$$\frac{w-x}{y} = \frac{w}{10} \Rightarrow 10w - 10x = yw \Rightarrow w = \frac{10x}{10-y}$$

$$\therefore \frac{dw}{dt} = \frac{(10-y)\left(10\frac{dx}{dt}\right) - (10x)\left(-\frac{dy}{dt}\right)}{(10-y)^2}.$$

When $x = 5$, $y = 6$: $\frac{dw}{dt} = \frac{(10-6)(10\cdot\frac{1}{2}) - (10\cdot 5)(-1)}{(10-6)^2} = \frac{35}{8}\ \frac{m}{s}$

$\therefore$ Shadow is moving along the ground at $\frac{35}{8}\ \frac{m}{s}$.