Question 1:

(a)[7 points] Determine the linearization (or linear approximation) to $f(x) = \dfrac{1}{\sqrt{1+3x}}$ at $a = 0$.

$$f(a) = f(0) = \frac{1}{\sqrt{1+3\cdot 0}} = 1$$

$$f'(x) = -\frac{1}{2}(1+3x)^{-\frac{3}{2}}(3)$$

$$f'(a) = f'(0) = -\frac{3}{2}.$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 1 - \frac{3}{2}(x-0)$$

$$= 1 - \frac{3}{2}x$$

$$\boxed{\therefore\ L(x) = 1 - \frac{3}{2}x}$$

(b)[3 points] Use your result from part (a) to estimate the value of $\dfrac{1}{\sqrt{1.03}}$.

$$\frac{1}{\sqrt{1.03}} = f(0.01) \approx L(0.01)$$

$$= 1 - \frac{3}{2}\left(\frac{1}{100}\right)$$

$$= 1 - \frac{3}{200}$$

$$= \boxed{\frac{197}{200}}$$

Question 2:

(a)[3 points] Differentiate $y = e^{x^5 - \ln(x^2)}$.

$$y' = e^{x^5 - \ln(x^2)} \left[5x^4 - \frac{1}{x^2}(2x)\right]$$

$$= \boxed{e^{x^5 - \ln(x^2)} \left[5x^4 - \frac{2}{x}\right]}$$

(b)[3 points] Determine $f'(0)$ if $f(x) = \ln\left(\frac{e^x}{x^2+1}\right)$.

$$f(x) = \ln\left(\frac{e^x}{x^2+1}\right) = \ln(e^x) - \ln(x^2+1)$$

$$= x - \ln(x^2+1)$$

$$\therefore \quad f'(x) = 1 - \frac{1}{1+x^2}(2x)$$

$$f'(0) = 1 - \frac{1}{1+0^2}(2 \cdot 0)$$

$$= \boxed{1}$$

(c)[4 points] Determine $g'(x)$ if $g(x) = 5^{\sqrt{1-x}} + \log_5 \sqrt{x}$.

$$g'(x) = 5^{\sqrt{1-x}} \ln 5 \ \tfrac{1}{2}(1-x)^{-\frac{1}{2}}(-1) + \frac{1}{\sqrt{x}\ln 5} \ \tfrac{1}{2} x^{-\frac{1}{2}}$$

$$= \boxed{\frac{-5^{\sqrt{1-x}} \ln 5}{2\sqrt{1-x}} + \frac{1}{2x \cdot \ln 5}}$$

Question 3:

(a)[5 points] Use logarithmic differentiation to determine y' where $y = \dfrac{(x-1)^7}{x^{2x}}$.

$$y = \frac{(x-1)^7}{x^{2x}}$$

$$\ln y = 7\ln(x-1) - 2x\ln x$$

$$\frac{1}{y}y' = \frac{7}{x-1} - 2\ln x - 2\cancel{x}\cdot\frac{1}{\cancel{x}}$$

$$\boxed{\therefore\ y' = \frac{(x-1)^7}{x^{2x}}\left[\frac{7}{x-1} - 2\ln x - 2\right]}$$

(b)[5 points] Determine the equation of the tangent line to curve $y + x^2e^y = 1 + \ln(x+3y)$ at the point $(1, 0)$.

$$\frac{d}{dx}\left[y + x^2e^y\right] = \frac{d}{dx}\left[1 + \ln(x+3y)\right]$$

$$y' + 2xe^y + x^2e^y y' = \frac{1}{x+3y}(1+3y')$$

at $x=1, y=0$:

$$y' + 2(1)e^0 + 1^2e^0y' = \frac{1}{1+3\cdot 0}(1+3y')$$

$$2y' + 2 = 1 + 3y'$$

$$y' = 1$$

$\therefore\ y - 0 = 1(x-1)$ or $\boxed{y = x-1}$

Question 4: This question deals with the function $f(x) = e^{-\frac{1}{2}x^2}$. Note that f has domain all real numbers.

(a) [**5 points**] Determine the intervals of increase and decrease of f and state the relative extrema, if any.

$$f'(x) = e^{-\frac{1}{2}x^2}\left(-\frac{1}{2}\cdot 2x\right)$$

$$= -e^{-\frac{1}{2}x^2}x$$

- $f'(x) = 0$? $x = 0$
- $f'(x)$ not exist? no such x.

crit. num:		0	
test values:	-1		1
$f'(x) = -e^{-\frac{1}{2}x^2}x$:	$+$	0	$-$
$f(x) = e^{-\frac{1}{2}x^2}$:	$\nearrow$	1	$\searrow$

$\therefore$ f is increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$.

f has a rel. max. of 1 at $x = 0$.

(b)[5 points] Determine the intervals of concavity of the graph of $y = f(x)$ and state the inflection points, if any.

$$f'(x) = -xe^{-\frac{1}{2}x^2}$$

$$f''(x) = -e^{-\frac{1}{2}x^2} - xe^{-\frac{1}{2}x^2}(-x)$$

$$= -e^{-\frac{1}{2}x^2}\left[1-x^2\right]$$

- $f''(x) = 0$?

$$1-x^2=0$$
$$(1-x)(1+x)=0$$
$$x=1,\quad x=-1$$

- $f''(x)$ not exist?
- $f(x)$ not defined?

$\Big\}$ no such x.

		-1		1	
test values:	-2		0		2
$f''(x) = -e^{-\frac{1}{2}x^2}(1-x^2)$:	$+$	0	$-$		$+$
$f(x) = e^{-\frac{1}{2}x^2}$:	$\smile$	$e^{-\frac{1}{2}}$	$\frown$	$e^{-\frac{1}{2}}$	$\smile$

$\therefore$ graph of $y=f(x)$ is concave up on $(-\infty,-1)$, $(1,\infty)$; concave down on $(-1,1)$.

Graph has inflection points at $(-1,e^{-\frac{1}{2}})$, $(1,e^{-\frac{1}{2}})$.

Question 5 [10 points]: Determine the absolute maximum and minimum values of $f(x) = (x^2 + 2x)^{1/3}$ on the interval $[-2, 2]$.

f is continuous and $[-2,2]$ is closed.

$$f'(x) = \frac{1}{3}(x^2+2x)^{-2/3}(2x+2)$$

$$= \frac{2(x+1)}{3\left[x(x+2)\right]^{2/3}}$$

- $f'(x) = 0$ at $x = -1$
- $f'(x)$ does not exist at $x = 0$, $x = -2$.

x	$f(x) = (x^2+2x)^{1/3}$
-2	0
-1	-1
0	0
2	2

$\therefore$ f has an abs. max. of 2 at $x = 2$;
f has an abs. min. of -1 at $x = -1$.