Math 121 S10N01 - Quiz 6 Name: SOLUTIONS.
Mar 17 2010 Stu#:

(1) [4 points] Determine the limit:

$$\lim_{x\to\infty} e^{-2x}\sin(x)$$

$$-1 \leq \sin(x) \leq 1$$

$$\therefore \quad -e^{-2x} \leq e^{-2x}\sin(x) \leq e^{-2x}.$$

Since $\lim_{x\to\infty} -e^{-2x} = 0 = \lim_{x\to\infty} e^{-2x}$, by the Squeeze Law,

$$\boxed{\lim_{x\to\infty} e^{-2x}\sin(x) = 0}$$

(2) [4 points] Differentiate:

$$y = \ln\left(e^{-x} + x^2 e^{-x}\right)$$

$$y = \ln\left[e^{-x}(1+x^2)\right]$$

$$= \ln(e^{-x}) + \ln(1+x^2)$$

$$= -x + \ln(1+x^2)$$

$$\boxed{\therefore \quad y' = -1 + \left(\frac{1}{1+x^2}\right)(2x)}$$

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(3) [7 points] Use logarithmic differentiation to determine the derivative:

$$y = (\csc x)^{\sqrt{x}}$$

$$\ln y = \ln\left[(\csc x)^{(x^{\frac{1}{2}})}\right]$$

$$\ln y = (x^{\frac{1}{2}})\ln[\csc x]$$

$$\therefore \frac{1}{y}\cdot y' = \left(\frac{1}{2}x^{-\frac{1}{2}}\right)\ln[\csc x] + (x^{\frac{1}{2}})\frac{1}{\cancel{\csc x}}\cdot(-\cancel{\csc x}\cdot\cot x)$$

$$\boxed{\therefore y' = (\csc x)^{\sqrt{x}}\left[\frac{1}{2}x^{-\frac{1}{2}}\cdot\ln(\csc x) - x^{\frac{1}{2}}\cot x\right]}$$