

(1) [6 points] Determine the linearization (or linear approximation) $L(x)$ of $f(x) = \cos x$ at $a = \pi/6$.

$$f(x) = \cos(x) ; f(a) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$f'(x) = -\sin(x); f'(a) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{6}\right)$$

(2) [9 points] Use a linear approximation to estimate $\sqrt[3]{7.9}$.

$$\text{Here } f(x) = x^{1/3}, a = 8$$

$$f(a) = 8^{1/3} = 2$$

$$f'(x) = \frac{1}{3}x^{-2/3}; f'(a) = \frac{1}{3}(8^{1/3})^{-2} = \frac{1}{12}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$

$$= 2 + \frac{1}{12}(x-8)$$

$$\therefore \sqrt[3]{7.9} \approx L(7.9) = 2 + \frac{1}{12}(7.9-8)$$

$$= 2 + \frac{1}{12}\left(-\frac{1}{10}\right)$$

$$= \frac{240-1}{120}$$

$$= \frac{239}{120}$$